新灯塔国际教育

AS 物理逆袭宝典

A-Level Physics • AS Level

Cambridge International AS & A Level Physics 9702 • AS Level

学生用书 Student Book

新灯塔国际教育

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写在前面

课堂上，常看到这样的情形：学生能听懂概念，方法也清楚，但一到自己动笔，就总在计算、单位换算和作图分析上出错。物理量纲、矢量分解、读图取斜率——这些基础步骤本应扎实，却常常成为失分的主要原因。问题往往不在“会不会想”，而在基本功还不够稳。

这本《AS 物理逆袭宝典》正是为这类同学准备的。全书紧扣 AS Level 物理考纲，把每章最核心的知识点拆解开，一步步练扎实。每章安排了三个层次：**Basic Drill（入门综合）**帮你建立手感；**Practice（综合练习）**巩固核心技能；**Advanced Practice（培优）**供有余力的同学进一步思考。目标很明确——把基础算对、算稳，为后面的学习铺好路。

基础暂时薄弱，不代表学不好 A Level。把基本功打扎实，后面的路会宽得多。

1 Physical quantities and units（物理量与单位）

Fundamental concepts of physical quantities, SI units, measurement uncertainties, and vectors.

Rules 规则

- All physical quantities consist of a numerical magnitude and a unit (物理量由数值和单位组成).
- The SI base units are kilogram (kg), metre (m), second (s), ampere (A), and kelvin (K) (国际单位制基本单位).
- An equation is homogeneous if every term has the same SI base units (方程的量纲一致性).
- Absolute uncertainties add when quantities are added or subtracted: $\Delta y = \Delta a + \Delta b$ (绝对不确定度相加).
- Percentage uncertainties add when quantities are multiplied or divided: $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$ (百分比不确定度相加).
- For powers, multiply the percentage uncertainty by the power: if $y = a^n$, then $\frac{\Delta y}{y} = n\frac{\Delta a}{a}$ (指数的不确定度).
- Scalars have magnitude only (标量只有大小), while vectors have both magnitude and direction (矢量有大小和方向).

Worked Examples 例题

Example 1.1

The drag force F acting on a sphere moving through a fluid is given by the equation $F = krv$, where r is the radius of the sphere and v is its velocity. Determine the SI base units of the constant k.

Solution

1. Express the equation in terms of units: $[F] = [k] \times [r] \times [v]$.
2. The SI base units of force F are kg m s^{-2}.
3. The SI base unit of radius r is m.
4. The SI base units of velocity v are m s^{-1}.
5. Rearrange to find the units of k: $[k] = \frac{[F]}{[r][v]} = \frac{\text{kg m s}^{-2}}{\text{m}\times\text{m s}^{-1}}$.
6. Simplify the expression: $[k] = \text{kg m}^{-1}\text{s}^{-1}$.

Example 1.2

A student measures the mass of a block as $m = 4.00 \pm 0.04$ kg and its volume as $V = 0.020 \pm 0.001$ m^3. Calculate the density of the block and its absolute uncertainty.

Solution

1. Calculate the density ρ: $\rho = \frac{m}{V} = \frac{4.00}{0.020} = 200$ kg m^{-3}.
2. Calculate the percentage uncertainty in mass: $\frac{0.04}{4.00} \times 100\% = 1.0\%$.
3. Calculate the percentage uncertainty in volume: $\frac{0.001}{0.020} \times 100\% = 5.0\%$.
4. Add the percentage uncertainties (since the quantities are divided): Total percentage uncertainty $= 1.0\% + 5.0\% = 6.0\%$.
5. Calculate the absolute uncertainty in density: $\Delta\rho = 200 \times 0.060 = 12$ kg m^{-3}.
6. State the final result: $\rho = 200 \pm 12$ kg m^{-3}.

Example 1.3

An aircraft is flying due north with a velocity of 150 m s^{-1} relative to the air. A wind is blowing from the west with a velocity of 40 m s^{-1}. Determine the magnitude and direction of the resultant velocity of the aircraft relative to the ground.

Solution

1. Identify the two perpendicular vector components: $v_y = 150$ m s^{-1} (North) and $v_x = 40$ m s^{-1} (East, since wind is from the West).
2. Use Pythagoras' theorem to find the magnitude of the resultant velocity v: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 150^2} = \sqrt{1600 + 22500} = \sqrt{24100} \approx 155$ m s^{-1}.
3. Use trigonometry to find the angle θ east of north: $\tan\theta = \frac{v_x}{v_y} = \frac{40}{150}$.
4. Calculate θ: $\theta = \tan^{-1}(0.267) \approx 14.9°$.
5. The resultant velocity is 155 m s^{-1} at an angle of 15° East of North.

Example 1.4

A micrometer screw gauge has a zero error of +0.03 mm. A student uses it to measure the diameter of a wire and records a reading of 1.25 mm. The student repeats the measurement several times and obtains a range of readings from 1.23 mm to 1.27 mm. Explain how the zero error affects the measurements and calculate the true diameter of the wire for the first reading.

Solution

1. A zero error is a type of systematic error. It causes all measurements to be shifted by the same amount in the same direction.
2. The random errors are indicated by the scatter of the readings (1.23 mm to 1.27 mm). This affects precision but not accuracy.
3. To correct for a positive zero error, subtract the zero error from the recorded reading.
4. True diameter = recorded reading − zero error = 1.25 mm − 0.03 mm = 1.22 mm.

考点总览 Key Points

- ☐ 我理解物理量 (physical quantity) 由数值和单位构成，并能对大纲内常见物理量进行合理估算 (estimate)。
- ☐ 我掌握五种 SI 基本量 (base quantity) 的单位，以及从 pico (p) 到 tera (T) 等常用词头 (prefix) 的换算。
- ☐ 我能将导出单位 (derived unit) 转化为基本单位，并用于检验物理方程的齐次性 (homogeneity)。
- ☐ 我熟悉系统误差 (systematic error，含零点误差) 与随机误差 (random error) 的特点及其对实验的影响。
- ☐ 我能准确区分测量结果中的准确度 (accuracy) 与精确度 (precision)。
- ☐ 我会通过绝对不确定度 (absolute uncertainty) 或百分比不确定度的简单相加评估导出量的不确定度。
- ☐ 我熟悉标量 (scalar) 与矢量 (vector) 的区别，并能列举大纲中常见的标量和矢量实例。
- ☐ 我会进行共面矢量 (coplanar vector) 的几何加法与减法运算。
- ☐ 我能将一个矢量正交分解 (resolve) 为两个互相垂直的分量（如 $F_x = F\cos\theta, F_y = F\sin\theta$）。

Basic Drill（入门综合）· 50

1. State the two essential parts that must be specified to define any physical quantity.

2. Convert a speed of 54 km h^{-1} into metres per second (m s^{-1}).

3. Estimate the kinetic energy of an athlete of mass 70 kg running at a top speed of 10 m s^{-1}.

4. Using the relation $F = ma$, express the SI base units of force (the newton, N).

5. Work done is given by $W = Fs$, where F is force and s is displacement. Express the SI base units of the joule (J).

6. Pressure is defined as force per unit area ($p = \frac{F}{A}$). Express the pascal (Pa) in terms of SI base units.

7. Power is defined as work done per unit time ($P = \frac{W}{t}$). Determine the SI base units of the watt (W).

8. Convert a pressure of 0.45 MPa into N cm^{-2}.

9. A liquid sample has a density of 1.25 g cm^{-3}. Convert this density to kg m^{-3}.

10. The drag force F acting on a sphere moving through a fluid is given by $F = k\rho v^2 A$, where ρ is the fluid density, v is the speed, A is the cross-sectional area, and k is a dimensionless constant. State the SI base units of each side of the equation to show that it is homogeneous.

11. The speed v of a transverse wave on a wire under tension T is given by $v = \sqrt{\frac{T}{\mu}}$, where μ is the mass per unit length of the wire. State the SI base units of $\frac{T}{\mu}$ and hence show that the equation is homogeneous.

12. State one difference between a systematic error and a random error in experimental measurements.

13. Distinguish between precision and accuracy in experimental measurements.

14. The current in a resistor is (2.0 ± 0.1) A and its resistance is (15.0 ± 0.3) Ω. Calculate the percentage uncertainty in the power dissipated in the resistor, given by $P = I^2 R$.

15. The length and width of a rectangular plate are measured as (20.0 ± 0.2) cm and (10.0 ± 0.1) cm respectively. Calculate the absolute uncertainty in the calculated area of the plate.

16. The time period T of an oscillating spring is measured to be (0.80 ± 0.02) s. Calculate the percentage uncertainty in the value of T^2.

17. A micrometer screw gauge has a zero error of -0.03 mm. The reading displayed when measuring the diameter of a metal wire is 2.45 mm. Determine the corrected diameter of the wire.

18. Classify each of the following quantities as either a scalar or a vector: electric current, momentum, kinetic energy, weight.

19. A crate is pulled across a horizontal floor by a rope. The tension in the rope is 120 N at an angle of 35° above the horizontal. Calculate the horizontal component of the tension force. Give your answer to 2 significant figures.

20. A projectile is launched with a horizontal velocity component of 24 m s^{-1} and a vertical velocity component of 10 m s^{-1}. Calculate the launch speed of the projectile.

21. Two horizontal forces of magnitude 9.0 N and 12.0 N act on an object at an angle of 90° to each other. Calculate the magnitude of the resultant force.

22. A rubber ball travels horizontally with a velocity of +8.0 m s^{-1} towards a wall, hits the wall, and rebounds along the same path with a velocity of −6.0 m s^{-1}. Calculate the magnitude of the change in velocity of the ball.

23. Electric potential difference V is defined as work done per unit charge ($V = \frac{W}{Q}$, where $Q = It$). Express the SI base units of the volt (V).

24. An electric current of 40 μA flows through a component for a time interval of 5.0 ms. Calculate the charge, in nanocoulombs (nC), that passes through the component.

25. Two forces, each of magnitude 50 N, act at a single point with an angle of 60° between them. Calculate the magnitude of the resultant force. Give your answer to 2 significant figures.

26. Convert a capacitance of 470 pF into units of microfarads (μF).

27. State a reasonable estimate for the wavelength of green light in a vacuum.

28. Express the SI derived unit of power, the watt (W), in terms of SI base units.

29. The period T of oscillation of a mass m attached to a spring of spring constant k is given by $T = 2\pi\sqrt{\frac{m}{k}}$. Show that the SI base units of $\sqrt{\frac{m}{k}}$ are seconds (s), given that the SI base units of k are kg s^{-2}.

30. A digital balance displays +0.15 g when no object is on the pan. State the type of experimental error caused by this fault, and describe its effect on all measured values.

31. A force of magnitude 65 N acts at an angle of 25° below the horizontal. Calculate the vertical component of this force.

32. A swimmer heads due north across a river with a speed of 1.2 m s^{-1} relative to the water. The river flows due east with a speed of 0.50 m s^{-1}. Calculate the magnitude of the resultant velocity of the swimmer.

33. The potential difference across a component is measured as $V = (12.0 \pm 0.6)$ V and the current is $I = (2.00 \pm 0.04)$ A. Calculate the percentage uncertainty in the resistance R, where $R = \frac{V}{I}$.

34. Electric charge Q is given by the relation $Q = It$, where I is electric current and t is time. Express the SI derived unit of electric charge, the coulomb ($extC$), in terms of SI base units.

35. From the list: acceleration, electric charge, energy, momentum, temperature, state all the vector quantities.

36. A ball moving horizontally to the right with a velocity of 8.0 m s^{-1} strikes a vertical wall and rebounds horizontally to the left with a velocity of 6.0 m s^{-1}. Taking the initial direction to the right as positive, calculate the change in velocity Δv of the ball.

37. A student records four readings for the diameter of a wire: 0.36 mm, 0.38 mm, 0.37 mm, and 0.37 mm. Determine the mean diameter and its absolute uncertainty, where the uncertainty is half the range of the readings.

38. Calculate the energy, in joules (J), equivalent to 2.5 kW h.

39. State a reasonable estimate for the speed of sound in air at room temperature.

40. The energy E of a photon is given by $E = hf$, where f is the frequency. Determine the SI base units of the Planck constant h.

41. The mass of an object is measured with a percentage uncertainty of 3%, and its speed is measured with a percentage uncertainty of 2%. Calculate the percentage uncertainty in its calculated kinetic energy $E_{\mathrm{k}} = \frac{1}{2}mv^2$.

42. Two coplanar forces of magnitudes 9.0 N and 12.0 N act on an object at right angles to each other. Calculate the angle between the resultant force and the 12.0 N force.

43. State what is meant by the precision of a set of experimental measurements.

44. The drag force F on a sphere of radius r moving at speed v through a fluid is given by $F = 6\pi\eta rv$. Determine the SI base units of the viscosity η.

45. State two methods that an experimenter can use to reduce the effect of random errors in an experiment.

46. Electric potential difference V has SI base units of kg $\mathrm{m^2 s^{-3} A^{-1}}$. Using the relationship $R = \frac{V}{I}$, express the SI derived unit of resistance, the ohm (Ω), in terms of SI base units.

47. A block of mass 4.0 kg is placed on a smooth slope inclined at 30° to the horizontal. Calculate the component of the weight of the block acting parallel to the slope.

48. The unstretched length of a wire is $L_1 = (1.500 \pm 0.002)$ m and its extended length is $L_2 = (1.535 \pm 0.002)$ m. Calculate the extension of the wire and its absolute uncertainty.

49. A student measures the time for 20 complete oscillations of a pendulum to be (36.0 ± 0.4) s. Calculate the period of one oscillation and its absolute uncertainty.

50. The radius r of a solid sphere is measured with a percentage uncertainty of 1.5%. Calculate the percentage uncertainty in the calculated volume $V = \frac{4}{3}\pi r^3$ of the sphere.

Practice（综合练习）· 25

1. Pressure is defined as force per unit area. Force is mass multiplied by acceleration. (a) Express the derived unit of force (the newton) in terms of SI base units. (b) Determine the SI base units of pressure.

2. The speed v of a transverse wave on a stretched string is given by the equation $v = \sqrt{\frac{T}{\mu}}$, where T is the tension in the string and μ is the mass per unit length of the string. (a) State the SI base units of v and T. (b) Show that the equation is homogeneous with respect to SI base units.

3. (a) Make a reasonable estimate of the mass, in kg, of an adult human. (b) Assuming the average density of a human is approximately equal to the density of water (1000 kg m^{-3}), estimate the volume, in m^3, of an adult human.

4. Two coplanar forces of magnitudes 45 N and 60 N act on a point object. The angle between the two forces is 90°. Calculate: (a) the magnitude of the resultant force, (b) the angle the resultant force makes with the 60 N force.

5. A metal cube has a mass of (120 ± 2) g and a side length of (2.50 ± 0.05) cm. (a) Calculate the density of the metal in g cm^{-3}. (b) Determine the percentage uncertainty in the density. (c) Calculate the absolute uncertainty in the density.

6. The viscous drag force F acting on a sphere of radius r moving with speed v through a fluid is given by $F = 6\pi\eta rv$, where η is a constant known as the viscosity of the fluid. (a) State the SI base units of F. (b) Determine the SI base units of η.

7. A student uses a micrometer screw gauge to measure the diameter of a wire. The micrometer has a zero error. The student takes five measurements at different positions along the wire. (a) Explain what is meant by a zero error and state whether it causes a random or a systematic error. (b) Explain how taking multiple measurements at different positions reduces the effect of random errors.

8. A car travels due north at a constant speed of 20 m s^{-1}. It then turns and travels due east at the same constant speed of 20 m s^{-1}. (a) State the difference between a scalar and a vector quantity. (b) Calculate the magnitude of the change in velocity of the car. (c) Determine the direction of the change in velocity.

9. The luminosity L (power emitted) of a spherical star is given by the equation $L = 4\pi r^2 \sigma T^4$ where r is the radius of the star, T is its thermodynamic temperature, and σ is the Stefan-Boltzmann constant. (a) Express the derived unit of power, the watt (W), in terms of SI base units. (b) Use the equation to determine the SI base units of the Stefan-Boltzmann constant σ. (c) For a particular star, the radius r is determined with a percentage uncertainty of 4.5% and the temperature T with a percentage uncertainty of 2.0%. Calculate the percentage uncertainty in the calculated luminosity L. (d) The temperature of the star is measured to be 6200 K. Calculate the absolute uncertainty in this temperature measurement.

10. A surveyor walks from point P to point Q, a distance of 150 m due East. They then walk from point Q to point R. The resultant displacement from P to R has a magnitude of 220 m at an angle of 40° North of due East. (a) Calculate the East component and the North component of the resultant displacement from P to R. (b) Determine the magnitude of the displacement from Q to R. (c) The 150 m distance from P to Q is measured with a percentage uncertainty of 2.0%. Calculate the absolute uncertainty in this measurement. (d) The surveyor's measuring device has a calibration fault, causing all recorded distances to be consistently 0.5 m larger than their true values. State the type of error this introduces, and state whether it affects the accuracy or the precision of the measurements.

11. A block is pulled along a horizontal surface by a string. The tension in the string is 85 N and it acts at an angle of 30° above the horizontal. (a) Calculate the horizontal component of the tension. (b) Calculate the vertical component of the tension.

12. Electrical power P is given by the equation $P = VI$, where V is the potential difference and I is the current. Power is defined as work done per unit time. (a) Determine the SI base units of power. (b) Use the equation to determine the SI base units of potential difference V.

13. Four students each measure the length of a table, which has a true length of 1.500 m. Student A records values of 1.50 m, 1.51 m, and 1.49 m. Student B records values of 1.652 m, 1.651 m, and 1.653 m. (a) Explain the distinction between precision and accuracy. (b) State and explain which student's measurements are more precise, and which are more accurate.

14. Kinetic energy E_k is given by the equation $E_k = \frac{1}{2}mv^2$. (a) Make a reasonable estimate of the mass m, in kg, of a fully grown horse. (b) Make a reasonable estimate of the maximum running speed v, in m s^{-1}, of a horse. (c) Use your estimates to calculate the maximum kinetic energy of the horse.

15. An aircraft is directed due north with a speed of 150 m s^{-1} relative to the air. A wind blows from the west to the east with a speed of 40 m s^{-1}. (a) Calculate the magnitude of the resultant velocity of the aircraft relative to the ground. (b) Calculate the angle of the resultant velocity measured east of due north.

16. The energy E of a photon is given by $E = \frac{hc}{\lambda}$, where h is the Planck constant, c is the speed of light, and λ is the wavelength. (a) State the SI base units of energy E. (b) Determine the SI base units of the Planck constant h.

17. The dimensions of a rectangular metal plate are measured as length $L = (24.0 \pm 0.2)$ cm and width $W = (15.0 \pm 0.1)$ cm. (a) Calculate the percentage uncertainty in L and in W. (b) Calculate the area of the plate in cm^2. (c) Determine the absolute uncertainty in the area.

18. A list of physical quantities is given: mass, displacement, temperature, velocity, kinetic energy, acceleration. (a) Define a vector quantity. (b) From the list, identify all the vector quantities. (c) From the list, identify all the scalar quantities.

19. (a) Make a reasonable estimate of the thickness, in cm, of a stack of 500 sheets of standard printer paper. (b) Use your estimate from (a) to determine the thickness of a single sheet of paper in micrometres (μm). (c) A student measures the thickness of a single sheet of paper using a micrometer screw gauge. Describe how the student can check whether the micrometer has a zero error before making the measurement.

20. The power P extracted by a wind turbine is given by the equation $P = \frac{1}{2}\rho A v^3$, where ρ is the density of the air, A is the circular area swept by the blades, and v is the wind speed. (a) State the SI base units of power P and density ρ. (b) Show that the equation is homogeneous with respect to SI base units. (c) During a test, the wind speed v is measured as 72 km h^{-1}. Convert this speed into m s^{-1}.

21. A ship travels 40 km on a bearing of 030° (measured clockwise from North), and then travels 60 km on a bearing of 120°. (a) Explain why these two displacements are perpendicular to each other. (b) Calculate the magnitude of the total displacement of the ship from its starting point. (c) Calculate the bearing of the final position from the starting point.

22. The pressure p of an ideal gas is given by the equation $p = \frac{1}{3}\rho\langle c^2\rangle$, where ρ is the density of the gas and $\langle c^2\rangle$ is the mean square speed of the gas molecules. (a) State the SI base units of ρ and $\langle c^2\rangle$. (b) Show that the equation is homogeneous with respect to base units.

23. Three coplanar forces act simultaneously on a small ring: a 40 N force acting vertically upwards, a 15 N force acting vertically downwards, and a 35 N force acting horizontally to the right. (a) Determine the magnitude and direction of the resultant vertical force. (b) Calculate the magnitude of the overall resultant force acting on the ring. (c) Determine the angle of the overall resultant force above the horizontal.

24. A projectile is fired from the ground with an initial velocity of 45 m s^{-1} at an angle of 60° to the horizontal. (a) Calculate the initial horizontal component of the velocity. (b) Calculate the initial vertical component of the velocity. (c) State whether the horizontal and vertical components are scalar or vector quantities.

25. The thermal energy Q required to change the temperature of a mass m of a substance by ΔT is given by $Q = mc\Delta T$, where c is the specific heat capacity. (a) State the SI base units of thermal energy Q and temperature change ΔT. (b) Determine the SI base units of specific heat capacity c.

Advanced Practice（培优）· 20

1. The rate of flow of a viscous fluid, Q, through a pipe is given by Poiseuille's equation: $Q = \frac{\pi r^4 \Delta p}{8\eta L}$. Q is the volume of fluid flowing per unit time. r is the radius of the pipe, L is its length, and Δp is the pressure difference between the ends of the pipe (pressure has SI base units kg $\mathrm{m^{-1}s^{-2}}$). η is the viscosity of the fluid. Determine the SI base units of η.

2. The acceleration of free fall g is determined using a simple pendulum. The period T is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum. The measured length $L = 0.850 \pm 0.002$ m and the measured period $T = 1.85 \pm 0.02$ s. Calculate g and its absolute uncertainty. Give your answer to an appropriate number of significant figures.

3. A car travels at a constant speed of 15 m $\mathrm{s^{-1}}$. It enters a roundabout heading due North and exits the roundabout heading due East, maintaining the same speed. Calculate the magnitude and the direction of the change in velocity of the car.

4. Estimate the mass of air in a typical school classroom. The density of air is approximately 1.2 kg $\mathrm{m^{-3}}$. State the estimated dimensions of the room used in your calculation.

5. A student uses a micrometer screw gauge to measure the diameter of a wire. The true diameter of the wire is known to be 1.25 mm. The student's readings are 1.41 mm, 1.42 mm, 1.40 mm, and 1.41 mm. (a) State and explain whether the student's readings are accurate. (b) State and explain whether the student's readings are precise. (c) Suggest a possible cause for this type of error.

6. The drag force F on an object moving through a fluid is given by $F = \frac{1}{2}C\rho A v^2$, where ρ is the fluid density, A is the cross-sectional area, and v is the velocity. Force F has SI base units kg m $\mathrm{s^{-2}}$. Determine the SI base units of the drag coefficient C.

7. A student measures the initial and final temperatures of a liquid using a thermometer with an absolute uncertainty of ± 0.5 °C. The initial temperature is 22.0 °C and the final temperature is 25.5 °C. Calculate the temperature rise and its percentage uncertainty.

8. A ship travels 45 km on a bearing of 040° and then 30 km on a bearing of 160°. (a) Determine the North and East components of the ship's final displacement from its starting position. (b) Calculate the magnitude of the final displacement.

9. The intensity of a sound wave is measured in microwatts per square centimetre (μW cm^{-2}). Power (measured in watts, W) has SI base units kg m^{2}s^{-3}. Express the unit μW cm^{-2} in terms of SI base units and a power of 10.

10. The kinetic energy E of a moving object is given by $E = \frac{1}{2}mv^2$. Estimate the kinetic energy of a fully loaded commercial passenger aircraft flying at cruising speed. State the estimated values of mass and speed used in your calculation.

11. A metal cylinder has a mass $m = 145 \pm 2$ g, length $L = 4.0 \pm 0.1$ cm, and diameter $d = 2.40 \pm 0.05$ cm. The density ρ is calculated using the equation $\rho = \frac{4m}{\pi d^2 L}$. Calculate the density in g cm^{-3} and its absolute uncertainty. Give your answer to an appropriate number of significant figures.

12. The speed v of a transverse wave on a stretched string is given by $v = k\sqrt{\frac{T}{\mu}}$, where T is the tension force in the string (SI base units kg m s^{-2}), μ is the mass per unit length of the string, and k is a dimensionless constant. (a) Determine the SI base units of μ. (b) Explain why checking the homogeneity of this equation cannot determine the value of k.

13. An aircraft flies with a velocity of 85 m s^{-1} due East relative to the air. A crosswind blows from the South with a velocity of 25 m s^{-1}. (a) Calculate the magnitude of the resultant velocity of the aircraft relative to the ground. (b) Determine the angle of the resultant velocity relative to East.

14. The electrical resistance R of a wire is given by $R = \frac{\rho L}{A}$. The resistivity $\rho = (1.6 \pm 0.1) \times 10^{-8}\ \Omega$ m, length $L = 2.50 \pm 0.05$ m, and cross-sectional area $A = (3.2 \pm 0.2) \times 10^{-7}$ m^2. Calculate the percentage uncertainty in the resistance R.

15. The specific heat capacity of water is 4.18 J $g^{-1}K^{-1}$. The joule (J) in SI base units is kg m^2s^{-2}. Express 4.18 J $g^{-1}K^{-1}$ in SI base units, using standard scientific notation.

16. A boat needs to travel directly across a river from point A to point B. The river flows due East with a speed of 1.5 m s^{-1}. The boat can travel at 4.0 m s^{-1} in still water. Point B is directly North of point A. Calculate the bearing on which the boat must be steered to reach point B directly.

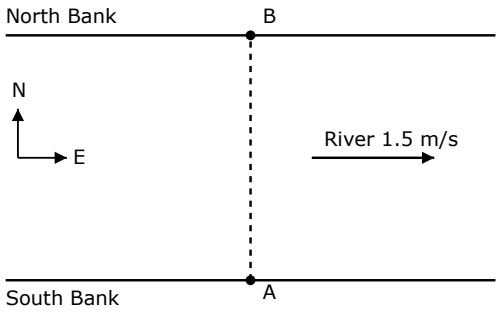

Fig. 16. Top-down view of the river crossing.

17. The speed of light c is 3.0×10^8 m s^{-1}. The frequency f and wavelength λ of an electromagnetic wave are related by $c = f\lambda$. Estimate the frequency of visible light. State the estimated wavelength used in your calculation.

18. A student investigates the relationship between two physical quantities, x and y. The theoretical relationship is $y = kx$, where k is a constant. The student plots a graph of y against x and obtains a straight line that does not pass through the origin, but instead has a positive y-intercept. (a) State the type of error that is indicated by the graph not passing through the origin. (b) Explain how the student can use the graph to determine the value of k despite this error.

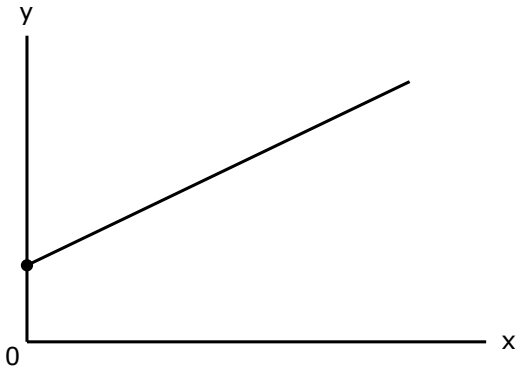

Fig. 18. Graph of y against x showing a straight line with a positive y-intercept.

19. A rectangular block has a measured width $w = 50 \pm 1$ mm and length $l = 80 \pm 2$ mm. (a) Calculate the maximum possible area of the rectangular face. (b) Use the maximum and minimum possible areas to determine the absolute uncertainty in the area.

20. The position vector $\mathbf{r}$ of a particle at time t is given by $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$, where $\mathbf{u}$ is the initial velocity vector and $\mathbf{a}$ is the acceleration vector. (a) Show that the equation is dimensionally homogeneous. (b) If $\mathbf{u}$ has a magnitude of 12 m s^{-1} due North and $\mathbf{a}$ has a magnitude of 5.0 m s^{-2} due East, calculate the magnitude of the displacement $\mathbf{r}$ at $t = 4.0$ s.

2 Kinematics（运动学）

Study of motion using equations of uniformly accelerated motion and graphical methods, including free fall and projectile motion.

Rules 规则

- Velocity is the rate of change of displacement (速度是位移的变化率), and acceleration is the rate of change of velocity (加速度是速度的变化率).
- For uniformly accelerated motion in a straight line, use the kinematic equations (匀加速直线运动方程): $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$, and $s = \frac{1}{2}(u + v)t$.
- The gradient of a displacement-time graph represents velocity (位移-时间图像的斜率表示速度).
- The gradient of a velocity-time graph represents acceleration (速度-时间图像的斜率表示加速度), and the area under the graph represents displacement (其下方的面积表示位移).
- Projectile motion can be resolved into two independent components: uniform velocity in one direction and uniform acceleration in a perpendicular direction (抛体运动可分解为两个独立的分量：一个方向上的匀速运动和垂直方向上的匀加速运动).
- In a uniform gravitational field without air resistance, all objects fall with the same constant acceleration $g = 9.81\ \text{m s}^{-2}$ (在没有空气阻力的均匀重力场中，所有物体以相同的恒定加速度下落).

Worked Examples 例题

Example 2.1

A car traveling at $15\ \text{m s}^{-1}$ accelerates uniformly at $2.5\ \text{m s}^{-2}$ for 4.0 s. Calculate the final speed and the distance traveled during this time.

Solution

Using the equation $v = u + at$, the final speed is $v = 15 + (2.5 \times 4.0) = 25\ \text{m s}^{-1}$. Using the equation $s = ut + \frac{1}{2}at^2$, the distance traveled is $s = (15 \times 4.0) + \frac{1}{2} \times 2.5 \times (4.0)^2 = 60 + 20 = 80\ \text{m}$.

Example 2.2

A velocity-time graph for a train shows a straight line from $v = 0$ at $t = 0$ to $v = 30$ m s^{-1} at $t = 20$ s. Determine the acceleration of the train and the displacement over the 20 s interval.

Solution

Acceleration is the gradient of the velocity-time graph: $a = \frac{\Delta v}{\Delta t} = \frac{30-0}{20-0} = 1.5$ m s^{-2}. Displacement is the area under the velocity-time graph, which forms a triangle: $s = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 20 \times 30 = 300$ m.

Example 2.3

A ball is thrown horizontally from a cliff of height 45 m with an initial speed of 12 m s^{-1}. Assuming no air resistance and $g = 9.81$ m s^{-2}, calculate the time taken to reach the ground and the horizontal distance traveled.

Solution

Consider the vertical motion with initial vertical velocity $u_y = 0$ and acceleration $a = 9.81$ m s^{-2}. Using $s = ut + \frac{1}{2}at^2$, we have $45 = 0 + \frac{1}{2} \times 9.81 \times t^2$. Solving for t gives $t = \sqrt{\frac{90}{9.81}} = 3.03$ s. For the horizontal motion, velocity is constant at $v_x = 12$ m s^{-1}. The horizontal distance is $s_x = v_x t = 12 \times 3.03 = 36.4$ m.

Example 2.4

In an experiment to determine the acceleration of free fall g, a steel ball is dropped from rest. It falls a distance of 0.850 m in a time of 0.416 s. Calculate the experimental value of g.

Solution

The ball falls from rest, so the initial velocity $u = 0$. Using the equation $s = ut + \frac{1}{2}at^2$, we substitute $s = 0.850$ m and $t = 0.416$ s. This gives $0.850 = 0 + \frac{1}{2} \times g \times (0.416)^2$. Rearranging for g yields $g = \frac{2 \times 0.850}{(0.416)^2} = \frac{1.70}{0.173} = 9.82$ m s^{-2}.

考点总览 Key Points

- ☐ 我理解并能区分路程 (distance)、位移 (displacement)、速率 (speed)、速度 (velocity) 与加速度 (acceleration) 的概念。
- ☐ 我会绘制位移-时间图 (s-t graph)，并能通过其斜率 (gradient) 计算物体的瞬时速度。
- ☐ 我掌握速度-时间图 (v-t graph)，能通过图线斜率求加速度、图线下方的面积求位移。
- ☐ 我能根据速度与加速度定义推导匀加速直线运动方程，如 $v = u + at$ 与 $s = ut + \frac{1}{2}at^2$。
- ☐ 我熟练运用运动学公式 (SUVAT equations) 求解匀加速直线运动问题，如 $v^2 = u^2 + 2as$。
- ☐ 我熟悉无空气阻力下物体在均匀重力场中做自由落体运动的规律与计算。
- ☐ 我能详细描述利用落体法测定重力加速度 (acceleration of free fall g) 的实验过程与数据分析。
- ☐ 我会分析一维匀速与垂直方向匀加速合成的二维抛体运动 (projectile motion)。

Basic Drill（入门综合）· 50

1. Define acceleration.

2. A car accelerates uniformly along a straight road from rest to a velocity of 24 m s^{-1} in a time of 6.0 s. Calculate the acceleration of the car.

3. A cyclist travels along a straight track at a constant velocity of 7.5 m s^{-1} for a duration of 16 s. Determine the displacement of the cyclist.

4. An aircraft accelerates from rest along a runway with a constant acceleration of 3.2 m s^{-2} for a time of 15 s. Calculate the distance travelled by the aircraft in this time.

5. The velocity–time graph shows the motion of a train travelling along a straight track. The train accelerates uniformly from rest to 16 m s^{-1} in 8.0 s, and then continues at a constant velocity of 16 m s^{-1} until $t = 20$ s. Determine the acceleration of the train during the first 8.0 s.

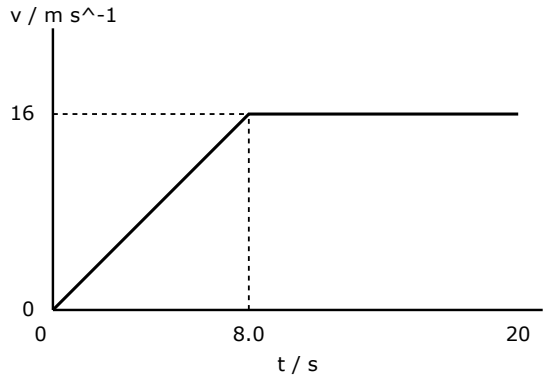

Fig. 5. Velocity–time graph for the motion of the train.

6. Using the velocity–time graph in Question 5, calculate the total displacement of the train between $t = 0$ and $t = 20$ s.

7. A steel sphere is released from rest and falls vertically downwards in a vacuum. Calculate the speed of the sphere after falling for 2.4 s.

8. A stone is dropped from rest into a vertical shaft. Air resistance is negligible. The stone hits the base of the shaft after a time of 1.8 s. Calculate the depth of the shaft.

9. A ball is projected vertically upwards with an initial speed of 15 m s^{-1}. Air resistance is negligible. Calculate the maximum height reached by the ball above its point of release.

10. A ball is projected vertically upwards with an initial speed of 15 m s^{-1}. Air resistance is negligible. Calculate the time taken for the ball to reach its maximum height.

11. A vehicle travelling at an initial velocity of 28 m s^{-1} decelerates uniformly to rest over a straight distance of 56 m. Calculate the magnitude of the deceleration.

12. The displacement–time graph for a particle moving in a straight line is a straight line passing through (0 s, 0 m) and (5.0 s, 35 m). State the physical quantity represented by the gradient of the graph, and calculate its value.

13. A runner completes one full lap around a circular running track of circumference 400 m in a time of 50 s, ending at the starting point. State: (a) the total distance travelled by the runner, (b) the magnitude of the overall displacement of the runner.

14. For the runner in Question 13 who completes the 400 m circular track in 50 s, calculate: (a) the average speed, (b) the magnitude of the average velocity.

15. A small ball is projected horizontally from the top of a vertical stand of height 1.8 m with an initial horizontal speed of 25 m s^{-1}. Air resistance is negligible. Calculate the time taken for the ball to reach the level ground.

16. A stone is thrown horizontally from the top of a vertical cliff of height 20 m with an initial speed of 18 m s^{-1}. Air resistance is negligible. Calculate the horizontal distance travelled by the stone before it hits the ground.

17. A marble is projected horizontally from a high table with an initial velocity of 12 m s^{-1}. Air resistance is negligible. Determine, at a time of 1.5 s after projection: (a) the horizontal component of velocity, (b) the vertical component of velocity.

18. State the kinematic equation linking initial velocity u, final velocity v, uniform acceleration a, and displacement s for linear motion.

19. In an experiment to determine the acceleration of free fall g, a small sphere is released from rest and falls through a vertical height h in time t. (a) State the relationship between h, g, and t when the initial velocity is zero. (b) State what quantity should be plotted on the vertical axis against t^2 on the horizontal axis to obtain a straight line with gradient equal to $\frac{1}{2}g$.

20. A student measures the time t for a steel ball to fall from rest through various vertical distances h. A graph of h against t^2 is plotted, giving a straight line of best fit through the origin with a gradient of 4.85 m s^{-2}. Determine the experimental value for the acceleration of free fall g.

21. A trolley moves in a straight line with uniform acceleration. Its speed increases from 3.0 m s^{-1} to 11.0 m s^{-1} over a time interval of 4.0 s. Calculate the distance travelled by the trolley during this time.

22. A tennis ball is dropped onto a hard floor and bounces vertically upwards. State what is represented by a sudden change in the sign of velocity on a velocity–time graph of the motion.

23. The velocity–time graph shows the motion of an object moving along a straight line. The object has an initial velocity of $+10$ m s^{-1} at $t = 0$, decelerates uniformly to 0 at $t = 4.0$ s, and then continues with the same constant acceleration to a velocity of -5.0 m s^{-1} at $t = 6.0$ s. Calculate the displacement of the object from $t = 0$ to $t = 6.0$ s.

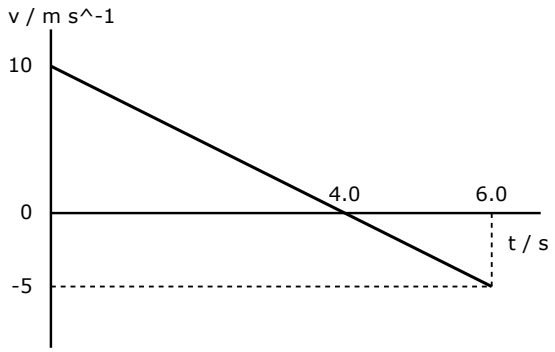

Fig. 23. Velocity–time graph showing motion with a change in direction.

24. A car starts from rest and accelerates uniformly at 2.5 m s^{-2} in a straight line for 6.0 s. It then travels at a constant velocity for a further 10.0 s. Calculate the total distance travelled by the car.

25. An aeroplane flies horizontally due East with a velocity of 120 m s^{-1} relative to the air. A steady crosswind blows due North at 35 m s^{-1}. Calculate the magnitude of the resultant velocity of the aeroplane.

26. Define displacement.

27. A train travels along a straight track with an initial velocity of 35 m s^{-1}. It decelerates uniformly at a rate of 1.4 m s^{-2} for a time of 15 s. Calculate the final velocity of the train.

28. State the physical quantity represented by the gradient of a displacement–time graph.

29. A small metal sphere is dropped from rest and strikes the ground with a speed of 14 m s^{-1}. Air resistance is negligible. Calculate the height from which the sphere was dropped.

30. An object moves along a straight line. Its velocity increases uniformly from 4.0 m s^{-1} to 12.0 m s^{-1} in a time of 5.0 s. Calculate the displacement of the object during this time.

31. A stone is released from rest at the top of a cliff of height 45 m. Air resistance is negligible. Calculate the time taken for the stone to reach the base of the cliff.

32. A small sphere is projected horizontally at a speed of 6.0 m s^{-1} from the edge of a bench of height 1.25 m. Air resistance is negligible. Calculate the horizontal distance travelled by the sphere before it strikes the floor.

33. A runner completes one lap of a 400 m oval running track in a time of 50 s, ending at the exact starting point. Determine (a) the average speed of the runner, and (b) the average velocity of the runner.

34. State the equation representing uniformly accelerated motion that relates displacement s, initial velocity u, final velocity v, and time t, without containing the acceleration a.

35. The variation with time t of the velocity v of a car moving along a straight road is shown in the figure. The car accelerates uniformly from rest to 16 m s^{-1} in 4.0 s, and then continues at a constant velocity until $t = 8.0$ s. Determine the acceleration of the car between $t = 0$ and $t = 4.0$ s.

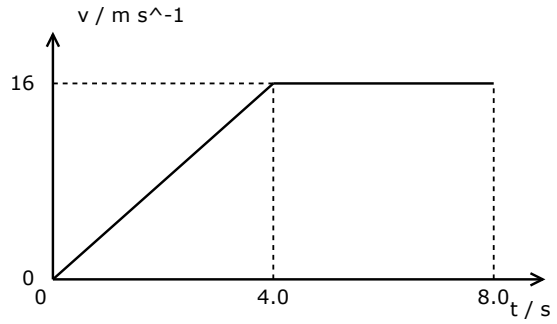

Fig. 35. Velocity-time graph for the motion of a car.

36. In an experiment using a light gate connected to a digital timer to determine the speed of a falling card, state the two quantities that must be measured to calculate the instantaneous speed of the card as it passes the gate.

37. A projectile is launched from the ground with an initial speed of 24 m s^{-1} at an angle of 30° above the horizontal. Calculate the horizontal component of the initial velocity.

38. A ball is projected at an angle of 40° above the horizontal. Air resistance is negligible. State the magnitude of the vertical component of its velocity when it reaches its maximum height.

39. State the key distinction between distance and displacement.

40. A vehicle accelerates uniformly along a straight road from an initial speed of 10 m s^{-1} to a final speed of 22 m s^{-1} over a distance of 80 m. Calculate the acceleration of the vehicle.

41. A car travelling along a straight road accelerates uniformly from an initial speed of 14 m s^{-1} to a speed of 26 m s^{-1} with an acceleration of 1.5 m s^{-2}. Calculate the time taken for the car to accelerate between these two speeds.

42. A small ball is thrown horizontally at 18 m s^{-1} from a window located 20 m vertically above flat ground. Air resistance is negligible. Calculate the time taken for the ball to reach the ground.

43. An object moves along a straight line. It has a displacement of +60 m in the first 6.0 s and a displacement of −20 m in the subsequent 4.0 s. Calculate the average velocity of the object over the entire 10.0 s period.

44. Describe the shape of the displacement–time graph for an object moving in a straight line with constant positive acceleration, starting from rest at the origin.

45. In a laboratory experiment to measure the acceleration of free fall g, explain why a dense steel sphere is preferred to a light plastic sphere of the same size.

46. A swimmer moves perpendicular to the bank of a river with a speed of 1.5 m s^{-1} relative to the water. The river flows downstream at a constant speed of 2.0 m s^{-1}. Calculate the magnitude of the resultant velocity of the swimmer relative to the bank.

47. A stone is dropped from rest into a deep vertical shaft and hits the bottom after a time of 3.2 s. Neglecting air resistance, calculate the depth of the shaft.

48. A cyclist accelerates uniformly along a straight track from a speed of 3.0 m s^{-1} to 9.0 m s^{-1} in a time of 4.0 s. Calculate the acceleration of the cyclist.

49. A car travelling at an initial velocity of 25 m s^{-1} decelerates uniformly at a rate of 5.0 m s^{-2} until it comes to rest. Calculate the stopping distance of the car.

50. An arrow is fired horizontally with an initial speed of 45 m s^{-1} from a height of 1.5 m above level ground. Air resistance is negligible. Calculate the speed of the arrow immediately before it hits the ground.

Practice（综合练习）· 25

1. (a) Define acceleration. (b) A car accelerates uniformly from a speed of 12 m s^{-1} to a speed of 28 m s^{-1} over a distance of 150 m. Calculate the time taken for this acceleration.

2. A train travels between two stations. It starts from rest and accelerates uniformly at 1.5 m s^{-2} for 12 s. It then travels at a constant velocity for 45 s before decelerating uniformly at 2.0 m s^{-2} to come to rest. Calculate: (a) the maximum velocity of the train. (b) the total distance travelled by the train.

3. A stone is dropped from rest from the top of a cliff. It takes 3.2 s to reach the ground below. Air resistance is negligible. Calculate: (a) the height of the cliff. (b) the velocity of the stone just before it hits the ground.

4. A ball is thrown vertically upwards with an initial speed of 14.0 m s^{-1} from a balcony that is 6.5 m above the ground. Air resistance is negligible. Calculate: (a) the maximum height reached by the ball above the ground. (b) the total time taken from when the ball is thrown until it hits the ground.

5. A small sphere rolls off the edge of a horizontal table with a speed of 2.4 m s^{-1}. The table is 1.05 m above the horizontal floor. Air resistance is negligible. Calculate: (a) the time taken for the sphere to reach the floor. (b) the horizontal distance from the edge of the table to the point where the sphere hits the floor.

6. A projectile is fired from horizontal ground with an initial speed of 32 m s^{-1} at an angle of 40° to the horizontal. Air resistance is negligible. Calculate: (a) the time of flight until the projectile returns to the ground. (b) the horizontal range of the projectile.

7. In an experiment to determine the acceleration of free fall g, a steel ball is released from rest from an electromagnet. It falls through a vertical distance of 0.850 m before hitting a trapdoor. The time of fall is recorded as 0.418 s. (a) Calculate the value of g obtained from these results. (b) Suggest one reason why the calculated value might be slightly less than the true value of g.

8. A cyclist accelerates uniformly from rest to a velocity of 8.0 m s^{-1} in a time of 6.0 s. She then maintains this velocity for 15 s before braking uniformly to a stop in 4.0 s. Calculate the acceleration during: (a) the first 6.0 s. (b) the final 4.0 s.

9. Fig. 9 shows the displacement-time graph for a moving object. Determine: (a) the velocity of the object at $t = 2.0$ s. (b) the velocity of the object at $t = 9.0$ s.

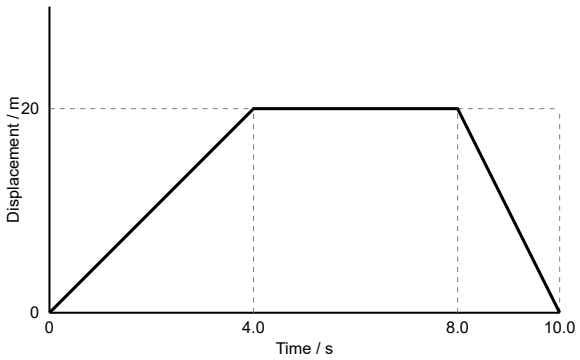

Fig. 9.

10. A football is kicked from ground level with an initial velocity of 22 m s^{-1} at an angle of 35° to the horizontal. A vertical wall of height 4.5 m is situated at a horizontal distance of 25 m from the point where the ball is kicked. Air resistance is negligible. Show whether the ball passes over the wall.

11. A car is travelling at a constant speed of 18 m s^{-1} along a straight road. As it passes a stationary police motorcycle, the motorcycle immediately starts to accelerate uniformly at 4.0 m s^{-2} in pursuit of the car. Calculate: (a) the time taken for the motorcycle to catch up with the car. (b) the distance travelled by the motorcycle when it catches the car.

12. A driver travelling at 28 m s^{-1} sees a hazard ahead. The driver's reaction time is 0.70 s. Once the brakes are applied, the car decelerates uniformly at 6.5 m s^{-2}. Calculate the total stopping distance of the car.

13. A model rocket is launched vertically upwards from rest. It accelerates uniformly at 18 m s^{-2} for 3.5 s. The engine then cuts out, and the rocket moves freely under gravity. Air resistance is negligible. Calculate: (a) the velocity of the rocket when the engine cuts out. (b) the maximum height reached by the rocket.

14. An aircraft lands on a runway with an initial speed of 65 m s^{-1}. It decelerates uniformly and comes to rest after travelling a distance of 1200 m. Calculate: (a) the deceleration of the aircraft. (b) the time taken to come to rest.

15. Define velocity.

16. A projectile is launched from horizontal ground with a velocity of 25 m s^{-1} at an angle of 55° above the horizontal. Air resistance is negligible. Calculate the speed of the projectile at a time 1.5 s after launch.

17. A runner runs 600 m due North in 120 s, and then runs 800 m due East in 180 s. Calculate for the whole journey: (a) the average speed. (b) the magnitude of the average velocity.

18. A stone is dropped from rest into a deep well. The sound of the stone hitting the water is heard 2.80 s after the stone is released. The speed of sound in air is 340 m s^{-1}. Air resistance on the stone is negligible. Calculate the depth of the well.

19. Fig. 19 shows the velocity-time graph for a particle moving in a straight line. Calculate: (a) the acceleration of the particle. (b) the total displacement of the particle from $t = 0$ to $t = 8.0$ s.

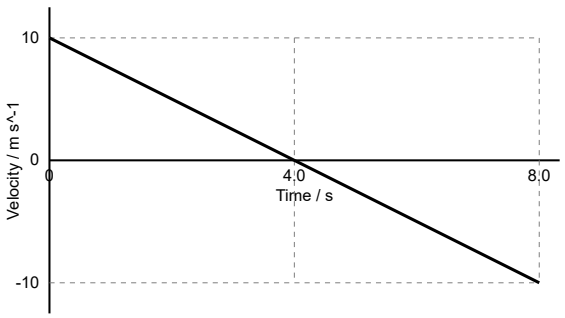

Fig. 19.

20. A dart is thrown horizontally with a speed of 9.5 m s^{-1} from a height of 1.8 m above horizontal ground. Air resistance is negligible. Calculate the angle to the horizontal at which the dart strikes the ground.

21. Object A is dropped from rest from a height of 40.0 m above the ground. At the same instant, object B is thrown vertically upwards from the ground with an initial speed of 25.0 m s^{-1}. The objects move along the same vertical line. Air resistance is negligible. Calculate the height above the ground at which the two objects collide.

22. An aeroplane flying horizontally at a constant speed of 140 m s^{-1} drops a supply package from an altitude of 850 m. Air resistance is negligible. Calculate the horizontal distance travelled by the package from the point of release to where it lands on the ground.

23. A block slides down a smooth inclined plane of length 3.2 m. It starts from rest at the top and reaches the bottom in 1.6 s. Calculate: (a) the acceleration of the block. (b) the velocity of the block as it reaches the bottom of the plane.

24. In a laboratory experiment, a piece of card of length 0.150 m is dropped vertically through a light gate. The light gate records a time of 0.042 s for the card to pass through. (a) Calculate the average speed of the card as it passes through the light gate. (b) A second light gate is positioned further down. The average speed of the card through the second light gate is 5.20 m s^{-1}. The time taken to travel between the centres of the two light gates is 0.166 s. Calculate the acceleration of the card.

25. A water balloon is thrown horizontally from a window at a speed of 6.0 m s^{-1}. It hits the ground 12 m horizontally from the window. Air resistance is negligible. Calculate: (a) the time the balloon is in the air. (b) the height of the window above the ground.

Advanced Practice（培优）· 20

1. (a) Use the definitions of velocity and acceleration to derive the equation $s = ut + \frac{1}{2}at^2$ for uniformly accelerated motion. (b) A particle passes point A with initial velocity u and constant acceleration a. It travels 40 m in the first 2.0 s and a further 60 m in the next 2.0 s. Determine u and a.

2. The figure shows the velocity-time graph for a train travelling between two stations. The total distance travelled by the train is 6.0 km. (a) Calculate the time interval T during which the train travels at constant velocity. (b) Calculate the average speed of the train for the whole journey.

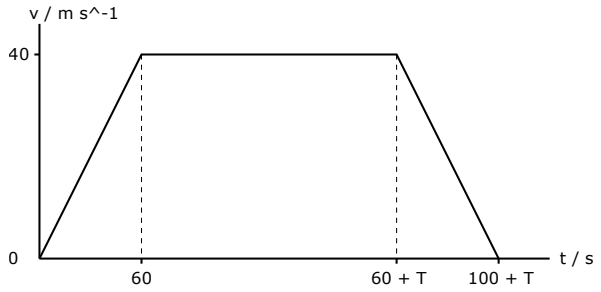

Fig. 2. Velocity-time graph for a train.

3. Stone A is dropped from rest from a bridge. 1.50 s later, stone B is thrown vertically downwards from the same position with an initial speed v. Both stones hit the water 45.0 m below at the same time. Air resistance is negligible. (a) Calculate the time taken for stone A to reach the water. (b) Calculate the initial speed v of stone B.

4. A ball is kicked horizontally with a speed of 18 m s^{-1} from the top of a vertical cliff of height 35 m. Air resistance is negligible. (a) Calculate the time taken for the ball to reach the sea below. (b) Calculate the magnitude and direction of the velocity of the ball just before it hits the sea.

5. A projectile is fired from level ground with an initial speed of 25.0 m s^{-1} at an angle of 40.0° to the horizontal. A vertical wall is located 45.0 m horizontally from the launch point. Air resistance is negligible. (a) Show that the height of the projectile when it passes over the wall is approximately 11 m. (b) Calculate the magnitude of the displacement of the projectile from the launch point when it reaches the highest point of its trajectory.

6. (a) Derive the equation $v^2 = u^2 + 2as$ for uniformly accelerated motion in a straight line. (b) A car is travelling at a constant speed of 28 m s^{-1}. The driver sees an obstacle and applies the brakes. The driver's reaction time is 0.60 s. The maximum constant deceleration the brakes can produce is 6.5 m s^{-2}. Calculate the minimum stopping distance of the car from the moment the driver sees the obstacle.

7. In an experiment to determine the acceleration of free fall g, a steel sphere is held by an electromagnet. The sphere is released and falls through a height h to a trapdoor. The timer starts when the current to the electromagnet is switched off. There is a small, constant time delay Δt before the sphere is actually released due to residual magnetism. The recorded time is t. (a) Show that $h = \frac{1}{2}g(t - \Delta t)^2$. (b) A graph of $\sqrt{h}$ on the y-axis against t on the x-axis is plotted. State expressions for the gradient and the y-intercept of the line.

8. Car A travels along a straight road at a constant velocity of 24 m s^{-1}. It passes a stationary police motorcycle B. At the exact moment car A passes, motorcycle B begins to accelerate uniformly at 4.0 m s^{-2} in pursuit. (a) Calculate the time taken for motorcycle B to catch car A. (b) Calculate the distance travelled by motorcycle B before it catches car A. (c) Calculate the velocity of motorcycle B at the instant it catches car A.

9. Water leaves the nozzle of a hosepipe at an angle of 35.0° above the horizontal with a speed of 14.0 m s^{-1}. The nozzle is situated at a vertical height of 1.20 m above level ground. Air resistance is negligible. (a) Calculate the maximum height above the ground reached by the water. (b) A vertical wall is located at a horizontal distance of 16.0 m from the nozzle. Calculate the time taken for the water to travel a horizontal distance of 16.0 m. (c) Calculate the vertical height above the ground at which the water hits the wall. (d) Determine the magnitude of the velocity of the water just before it hits the wall.

10. A hot air balloon is rising vertically at a constant velocity of 5.0 m s^{-1}. A sandbag is released from the balloon when it is at a height of 60 m above the ground. Air resistance is negligible. (a) State the initial velocity of the sandbag relative to the ground. (b) Calculate the maximum height above the ground reached by the sandbag. (c) Calculate the time taken for the sandbag to reach the ground from the moment it is released.

11. A tennis player serves a ball horizontally from a height of 2.40 m with a speed of 26.0 m s^{-1}. The net is 11.9 m away horizontally and has a height of 0.91 m. Air resistance is negligible. (a) Show that the ball clears the net, and determine the vertical clearance height. (b) Calculate the horizontal distance from the server to the point where the ball lands on the ground. (c) Calculate the magnitude of the velocity of the ball just before it hits the ground.

12. A ball is thrown from an origin $(0, 0)$ with initial speed u at an angle θ above the horizontal. The ball passes through the coordinates $(x_1, y_1) = (12.0\text{ m}, 4.50\text{ m})$ and $(x_2, y_2) = (24.0\text{ m}, 0.0\text{ m})$. Air resistance is negligible. (a) Explain why the time taken to travel from $x = 0$ to $x = 12.0$ m is equal to the time taken to travel from $x = 12.0$ m to $x = 24.0$ m. (b) Determine the horizontal component of the initial velocity. (c) Determine the angle θ.

13. Car A travels at a constant velocity of 18.0 m s^{-1} along a straight road. Car B is travelling in the same direction at 26.0 m s^{-1}. At time $t = 0$, Car B is 40.0 m behind Car A, and the driver of Car B applies the brakes to produce a uniform deceleration of 0.500 m s^{-2}. (a) Calculate the time at which the two cars have the same velocity. (b) Calculate the distance travelled by Car A from $t = 0$ to the instant they have the same velocity. (c) Calculate the time t at which Car B first draws level with Car A. (d) Calculate the velocity of Car B at the instant it draws level with Car A.

14. Projectile A is launched horizontally from the top of a tower of height 45.0 m with a speed of 12.0 m s^{-1}. At the same instant, Projectile B is launched vertically upwards from the ground at a horizontal distance of 24.0 m from the base of the tower. The two projectiles collide in mid-air. Air resistance is negligible. (a) Calculate the time taken from launch for the projectiles to collide. (b) Calculate the initial upward speed of projectile B.

15. A steel ball is released from rest from the top of a tall building. Exactly 1.20 s later, a second identical ball is released from rest from the same position. Air resistance is negligible. (a) Explain why, once both balls are in the air, the vertical distance between them increases at a constant rate. (b) Calculate the time elapsed, from the release of the first ball, for the vertical distance between the two balls to become 25.0 m. (c) Calculate the vertical distance fallen by the first ball in this time.

16. A stone is dropped from rest into a deep well. The sound of the stone hitting the water is heard 3.40 s after it was dropped. The speed of sound in air is 340 m s^{-1}. Air resistance on the stone is negligible. (a) Write an expression for the depth d of the well in terms of the time t_1 taken for the stone to fall. (b) Determine the depth of the well.

17. A ball is launched from level ground with an initial speed of 22.0 m s^{-1} at an angle of 60.0° to the horizontal. It hits a vertical wall that is situated at a horizontal distance of 25.0 m from the launch point. Air resistance is negligible. (a) Calculate the time taken for the ball to reach the wall. (b) Calculate the height above the ground at which the ball hits the wall. (c) Determine whether the ball is travelling upwards or downwards at the instant it hits the wall, showing your working.

18. A stone is thrown from the edge of a cliff with an initial speed u at an angle of 30.0° above the horizontal. The stone reaches a maximum height of 4.60 m above the level of the cliff edge. The stone hits the sea at a horizontal distance of 65.0 m from the cliff edge. Air resistance is negligible. (a) Show that the initial speed u of the stone is 19.0 m s^{-1}. (b) Calculate the total time of flight of the stone. (c) Calculate the vertical height of the cliff edge above the sea. (d) Calculate the magnitude of the displacement of the stone from its launch point to the point where it hits the sea.

19. A projectile is fired horizontally with a speed of 15.0 m s^{-1} from the top of a straight slope. The slope makes an angle of 30.0° downwards from the horizontal. Air resistance is negligible. (a) Write equations for the horizontal distance x and the vertical distance y fallen by the projectile in time t. (b) Calculate the time of flight until the projectile hits the slope. (c) Calculate the straight-line distance from the launch point to the impact point on the slope.

20. In an experiment to determine g, a student drops a piece of card vertically through two light gates. The card has a vertical length of 0.150 m. The first light gate records a time of 0.075 s for the card to pass through. The second light gate records a time of 0.037 s. The vertical distance between the centres of the two light gates is 0.650 m. (a) Calculate the average speed of the card as it passes through each light gate. (b) Assuming these average speeds are equal to the instantaneous speeds at the centre of each light gate, calculate the acceleration of free fall g.

3 Dynamics（动力学）

Non-uniform motion with drag forces, terminal velocity, and the principle of conservation of linear momentum in one and two dimensions.

Rules 规则

- Drag forces, such as air resistance, increase as the speed of an object increases (阻力随速度增加而增大).
- Terminal velocity is reached when the resistive force equals the driving force or weight, resulting in zero acceleration (当阻力等于驱动力或重力时，加速度为零，达到终端速度).
- The principle of conservation of momentum states that the total momentum of a closed system remains constant provided no external resultant force acts ($p = mv$, 动量守恒定律).
- In a perfectly elastic collision, total kinetic energy is conserved and the relative speed of approach equals the relative speed of separation ($u_1 - u_2 = v_2 - v_1$, 弹性碰撞中动能守恒且接近速度等于分离速度).
- In an inelastic collision, momentum is conserved but some kinetic energy is transferred to other forms (非弹性碰撞中动量守恒，但动能不守恒).
- For two-dimensional collisions, momentum is conserved independently in both the x and y directions (二维碰撞中，动量在各个方向上独立守恒).

Worked Examples 例题

Example 3.1

A skydiver falls from a stationary balloon. Describe and explain qualitatively the variation of the acceleration of the skydiver from the moment they step out of the balloon to the time before the parachute is opened.

Solution

Initially, the skydiver's speed is zero, so air resistance is zero and acceleration is g downwards. As speed increases, air resistance increases. The resultant downward force decreases, so acceleration decreases. Eventually, air resistance equals weight. The resultant force is zero, so acceleration is zero and the skydiver reaches a constant terminal velocity.

Example 3.2

Trolley A of mass 0.80 kg moves at 2.5 m s^{-1} and collides with a stationary trolley B of mass 1.2 kg. They stick together after the collision. Calculate their common velocity after the collision.

Solution

By the principle of conservation of momentum: $m_A u_A + m_B u_B = (m_A + m_B)v$. Substituting the values: $0.80 \times 2.5 + 1.2 \times 0 = (0.80 + 1.2)v$. $2.0 = 2.0v$. Therefore, $v = 1.0$ m s^{-1}.

Example 3.3

Sphere X of mass m moving at 4.0 m s^{-1} collides head-on with sphere Y of mass $2m$ moving at 1.0 m s^{-1} in the same direction. The collision is perfectly elastic. Determine the velocity of sphere X after the collision.

Solution

For an elastic collision, relative speed of approach equals relative speed of separation: $u_X - u_Y = v_Y - v_X$. $4.0 - 1.0 = v_Y - v_X$, so $v_Y = v_X + 3.0$. By conservation of momentum: $m(4.0) + 2m(1.0) = mv_X + 2mv_Y$. Dividing by m: $4.0 + 2.0 = v_X + 2v_Y$. $6.0 = v_X + 2(v_X + 3.0) = 3v_X + 6.0$. $3v_X = 0$. Therefore, $v_X = 0$ m s^{-1}.

Example 3.4

A particle of mass m travelling at velocity v in the x-direction collides with a stationary particle of mass m. After the collision, the first particle moves at velocity v_1 at an angle of 30° above the x-axis, and the second particle moves at velocity v_2 at an angle of 60° below the x-axis. Show that $v_1 = v\cos 30°$.

Solution

Conserve momentum in the y-direction: $0 = mv_1 \sin 30° - mv_2 \sin 60°$. Therefore, $v_2 = v_1 \frac{\sin 30°}{\sin 60°} = v_1 \frac{1/2}{\sqrt{3}/2} = \frac{v_1}{\sqrt{3}}$. Conserve momentum in the x-direction: $mv = mv_1 \cos 30° + mv_2 \cos 60°$. Substitute v_2: $v = v_1 \frac{\sqrt{3}}{2} + \left(\frac{v_1}{\sqrt{3}}\right)\frac{1}{2} = v_1\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}\right) = v_1 \frac{4\sqrt{3}}{6} = v_1 \frac{2\sqrt{3}}{3}$. Since $\cos 30° = \frac{\sqrt{3}}{2}$, we have $v = v_1 \frac{2}{\sqrt{3}} = \frac{v_1}{\cos 30°}$. Thus, $v_1 = v\cos 30°$.

考点总览 Key Points

- ☐ 我理解摩擦力 (frictional force) 与流体阻力 (drag force)，知道阻力随速率增加而增大
- ☐ 我能定性分析物体在考虑空气阻力 (air resistance) 的重力场中的变速下落过程
- ☐ 我掌握终端速度 (terminal velocity) 的物理意义以及物体达到该速度时的平衡条件
- ☐ 我能准确表述动量守恒定律 (principle of conservation of momentum) 及其成立条件
- ☐ 我会应用动量守恒定律解决一维 (1D) 碰撞与相互作用问题（$m_1u_1+m_2u_2=m_1v_1+m_2v_2$）
- ☐ 我掌握通过正交分解或矢量图解决二维 (2D) 相互作用中的动量守恒计算
- ☐ 我理解弹性碰撞 (elastic collision) 中系统总动能 (kinetic energy) 守恒的特征
- ☐ 我能运用弹性碰撞中接近相对速度等于分离相对速度的规律（$u_1-u_2=v_2-v_1$）
- ☐ 我熟悉非弹性碰撞 (inelastic collision) 中动量依然守恒但总动能不守恒的性质

Basic Drill（入门综合）· 50

1. Define linear momentum.

2. A tennis ball of mass 0.058 kg travels at a velocity of 35 m s^{-1}. Calculate the magnitude of the linear momentum of the ball.

3. State the principle of conservation of momentum.

4. A package of mass 2.50 kg falls vertically through the air at its terminal velocity. Calculate the magnitude of the air resistance force acting on the package.

5. Trolley A moves along a straight track at 4.0 m s^{-1} and collides elastically with trolley B, which is moving at 1.5 m s^{-1} in the same direction. State the relative speed of separation of the two trolleys after the collision.

6. A trolley of mass 0.60 kg travelling at a speed of 2.5 m s^{-1} collides with a stationary trolley of mass 0.40 kg. The two trolleys stick together upon impact. Calculate their common speed after the collision.

7. A skydiver steps out of an aircraft. Explain qualitatively what happens to the acceleration of the skydiver as their downward speed increases before opening the parachute.

8. State what is meant by an elastic collision in terms of kinetic energy.

9. A rifle of mass 3.2 kg fires a bullet of mass 0.016 kg horizontally at a speed of 450 m s^{-1}. Calculate the magnitude of the recoil speed of the rifle.

10. State how the drag force acting on an object moving through a fluid changes as the speed of the object increases.

11. Ball P of mass 0.20 kg moves at +3.0 m s^{-1} and collides head-on elastically with an identical ball Q of mass 0.20 kg moving at −2.0 m s^{-1}. Determine the velocity of ball P immediately after the collision.

12. State the magnitude of the resultant force on a raindrop when it is falling at its terminal velocity.

13. Two blocks move towards each other on a smooth horizontal surface. Block X has mass 1.5 kg and speed 4.0 m s^{-1} to the right. Block Y has mass 2.0 kg and speed 3.0 m s^{-1} to the left. Taking the direction to the right as positive, calculate the total linear momentum of the system.

14. A small sphere falls vertically through a column of oil. At a particular instant, the weight of the sphere is 0.15 N and the upward viscous drag force is 0.09 N. Calculate the magnitude of the resultant force acting on the sphere.

15. A car of mass 1200 kg moving at 15 m s^{-1} collides with a stationary car of mass 800 kg. After the collision, the 1200 kg car continues forward in the same direction at 7.0 m s^{-1}. Calculate the speed of the 800 kg car immediately after the collision.

16. A stationary object of mass 0.50 kg explodes into two pieces. One piece of mass 0.20 kg moves west at a speed of 30 m s^{-1}. Determine the velocity of the second piece.

17. Block 1 moves with velocity +6.0 m s^{-1} and Block 2 moves with velocity +2.0 m s^{-1} along the same straight line. Calculate the relative speed of approach of the two blocks.

18. State the feature of a velocity – time graph that indicates an object has reached its terminal velocity.

19. A stationary particle at the origin decays into two fragments of masses m and $2m$. The fragment of mass m moves along the positive x-axis with linear momentum $+p$. State the linear momentum of the fragment of mass $2m$.

20. During an inelastic collision between two vehicles, state whether total momentum is conserved and whether total kinetic energy is conserved.

21. A rubber ball of mass 0.15 kg hits a vertical wall normally at a speed of 12 m s^{-1} and rebounds elastically along its original path at 12 m s^{-1}. Calculate the magnitude of the change in momentum of the ball.

22. State the acceleration of a small steel ball when it falls at terminal velocity through a cylinder of liquid.

23. An object travelling with momentum entirely in the x-direction breaks into two pieces. One piece has a momentum component of +3.0 kg m s^{-1} in the y-direction. Determine the momentum component of the other piece in the y-direction.

24. A solid metal sphere is released from rest in air. State the initial magnitude of the drag force acting on the sphere at the exact instant of release.

25. Two objects of masses m_1 and m_2 with initial velocities u_1 and u_2 collide along a straight line and have final velocities v_1 and v_2. State the mathematical equation representing the conservation of linear momentum for this collision.

26. A space probe of mass 450 kg travels through deep space at a constant velocity of 1200 m s^{-1}. Calculate the magnitude of the linear momentum of the space probe.

27. An object falls vertically through air and reaches terminal velocity. State the magnitude of the resultant force acting on the object at terminal velocity.

28. Trolley A moves at a velocity of 6.0 m s^{-1} along a straight track towards trolley B, which is travelling in the same direction at 2.0 m s^{-1}. The two trolleys collide elastically. Calculate the relative speed of separation of the trolleys after the collision.

29. A trolley of mass 0.40 kg travelling at +3.0 m s^{-1} collides head-on with a trolley of mass 0.20 kg travelling at −1.5 m s^{-1}. The trolleys stick together on impact. Calculate the common velocity of the trolleys after the collision.

30. A puck on a frictionless horizontal air table travels with momentum purely along the positive x-axis. It collides with a second, stationary puck. Immediately after the collision, the first puck has a momentum component of $+1.6$ kg m s^{-1} along the y-axis. Determine the momentum component of the second puck along the y-axis.

31. State whether total kinetic energy is conserved in a perfectly elastic collision.

32. A stationary rifle of mass 4.0 kg fires a bullet of mass 0.020 kg horizontally with a velocity of 300 m s^{-1}. Calculate the recoil speed of the rifle.

33. A skydiver is falling at terminal velocity. The parachute is then opened. State the direction of the resultant force on the skydiver immediately after the parachute opens.

34. A trolley of mass 0.50 kg moves at a velocity of 2.0 m s^{-1} towards a stationary trolley of mass 0.50 kg. Calculate the total initial linear momentum of the system.

35. State the key difference regarding total kinetic energy between an elastic collision and an inelastic collision.

36. Explain why an object released from rest in the atmosphere accelerates at first, but eventually reaches a constant terminal velocity.

37. A stationary object explodes into two fragments of mass 2.0 kg and 3.0 kg. The 2.0 kg fragment moves due North at a speed of 6.0 m s^{-1}. Determine the magnitude and direction of the velocity of the 3.0 kg fragment.

38. Trolley P travels at 5.0 m s^{-1} towards a stationary trolley Q. The collision is elastic. Immediately after the collision, trolley P rebounds in the opposite direction at 1.0 m s^{-1}. Calculate the speed of trolley Q after the collision.

39. A steel ball is released from rest in a tall cylinder of oil. Describe what happens to the acceleration of the ball as it falls towards its terminal velocity.

40. Block X of mass 1.2 kg travelling at 4.0 m s^{-1} collides with a stationary block Y of mass 0.80 kg. After the collision, block X continues in its initial direction at 1.0 m s^{-1}. Calculate the velocity of block Y after the collision.

41. State the necessary condition for the total momentum of a system of interacting particles to remain constant.

42. A solid metal sphere and an identical-sized hollow plastic sphere are dropped from rest from the same height in air. State which sphere achieves a greater terminal velocity.

43. A particle of mass 1.5 kg moving at 8.0 m s^{-1} in the positive x-direction collides with a stationary particle of mass 2.5 kg. Calculate the total momentum of the system after the collision in the x-direction.

44. Two identical gliders collide and stick together on a frictionless air track. State what happens to the total momentum and the total kinetic energy of the gliders during the collision.

45. A small ball falls vertically through a viscous liquid at a constant velocity. Name the downward force acting on the ball and the upward resistive force opposing its motion.

46. Two identical billiard balls move directly towards each other with speeds of 4.0 m s^{-1} and 2.0 m s^{-1} respectively. The collision between them is perfectly elastic. Calculate the relative speed of separation of the balls after the collision.

47. A model rocket of mass 0.80 kg is at rest in space. It ejects 0.050 kg of exhaust gas backwards at a velocity of 120 m s^{-1}. Calculate the forward velocity of the remaining 0.75 kg rocket body.

48. A car accelerates from 10 m s^{-1} to 20 m s^{-1} on a horizontal road. State whether the resistive force of air resistance increases, decreases, or remains constant.

49. Trolley P of mass 2.0 kg moving at +3.0 m s^{-1} collides with trolley Q of mass 1.0 kg moving at −2.0 m s^{-1}. After the collision, trolley P moves at +1.0 m s^{-1}. Calculate the final velocity of trolley Q.

50. Define what is meant by an inelastic collision.

Practice（综合练习）· 25

1. A sphere is released from rest in a tall cylinder of oil. Describe and explain qualitatively the variation of the acceleration of the sphere as it falls, until it reaches terminal velocity.

2. Trolley A of mass 0.80 kg moves at 2.5 m s^{-1} towards a stationary trolley B of mass 1.2 kg. They collide and stick together. Calculate the velocity of the combined trolleys immediately after the collision.

3. A sphere X of mass 0.40 kg travelling at 3.0 m s^{-1} collides with a stationary sphere Y of mass 0.60 kg. After the collision, sphere X moves in the opposite direction at 0.60 m s^{-1} and sphere Y moves in the original direction of X at 2.4 m s^{-1}. Show that the collision is perfectly elastic.

4. Object P of mass 2.0 kg moving at 4.0 m s^{-1} collides head-on with object Q of mass 3.0 kg moving at 1.0 m s^{-1} in the same direction. The collision is perfectly elastic. Let the final velocities of P and Q be v_P and v_Q respectively. (a) State the relative speed of separation of the two objects. (b) Calculate v_P and v_Q.

5. A particle A of mass 0.030 kg travels at 5.0 m s^{-1} in the $+x$ direction. It collides with a stationary particle B of mass 0.040 kg. After the collision, particle A moves at 3.0 m s^{-1} in the $+y$ direction. Calculate the magnitude of the momentum of particle B after the collision.

6. A skydiver falls from a stationary balloon. The skydiver reaches a terminal velocity v_1. The skydiver then opens a parachute and reaches a new terminal velocity v_2. Explain why v_2 is less than v_1.

7. A stationary explosive device of mass 0.25 kg explodes into two fragments. Fragment A of mass 0.10 kg moves at 45 m s^{-1} to the left. Calculate the magnitude and state the direction of the velocity of fragment B immediately after the explosion.

8. A lump of clay of mass 0.25 kg travelling at 12 m s^{-1} collides with a stationary block of wood of mass 1.5 kg. The clay sticks to the wood. (a) Calculate the speed of the combined mass immediately after the collision. (b) Calculate the loss in total kinetic energy during the collision.

9. Object X of mass 2.0 kg moves at 6.0 m s^{-1} along a horizontal surface and collides with a stationary object Y of mass 3.0 kg. After the collision, X moves at 3.0 m s^{-1} at an angle of 60° to its original path. (a) Calculate the component of the momentum of Y perpendicular to the original path of X. (b) Calculate the component of the momentum of Y parallel to the original path of X.

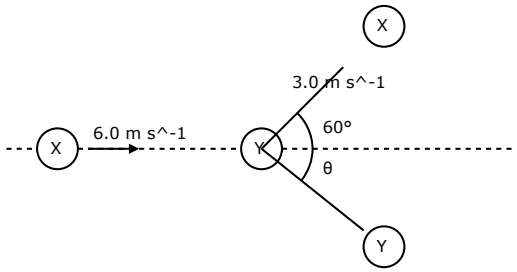

Fig. 9. Diagram showing the paths of objects X and Y before and after the collision.

10. A small metal sphere is falling through a liquid. The viscous drag force F_D on the sphere is given by $F_D = kv$, where v is the speed and k is a constant. The weight of the sphere is 4.5×10^{-3} N and the upthrust is negligible. The terminal velocity of the sphere is 0.055 m s^{-1}. Calculate the constant k and state its SI base units.

11. A ball of mass 0.15 kg moves at 8.0 m s^{-1} and strikes a rigid vertical wall normally. The collision is perfectly elastic. (a) State the speed of the ball immediately after the collision. (b) Calculate the magnitude of the change in momentum of the ball.

12. A toy train engine of mass M travelling at 0.60 m s^{-1} collides with a stationary carriage of mass 0.40 kg. They couple together and move off with a speed of 0.24 m s^{-1}. Calculate the mass M of the engine.

13. A stone is thrown vertically upwards in the air. Air resistance is not negligible. Compare the time taken for the stone to reach its maximum height with the time taken to fall back to its starting position. Explain your reasoning in terms of the forces acting on the stone.

14. Sphere A of mass 2.0 kg moves at 4.0 m s^{-1} and collides with stationary sphere B of mass 3.0 kg. Immediately after this collision, A is stationary and B moves towards a stationary sphere C of mass 1.5 kg. B and C collide and stick together. Calculate the final velocity of the combined spheres B and C.

15. A disc A of mass m sliding on a frictionless horizontal surface at 5.0 m s^{-1} collides with a stationary identical disc B. After the collision, disc A moves at 3.0 m s^{-1} at an angle θ to its original direction, and disc B moves at 4.0 m s^{-1} at an angle ϕ to the original direction of A. Determine whether the collision is elastic.

16. A puck A of mass 0.20 kg travelling at 8.0 m s^{-1} collides with a stationary puck B of the same mass. After the collision, puck A moves at an angle of 30° to its original path, and puck B moves at an angle of 60° to the original path of A, on the opposite side of the original path. Calculate the speed of puck A after the collision.

17. A curling stone of mass 19 kg slides on frictionless ice at a speed of 1.5 m s^{-1} and collides with a stationary identical stone. After the collision, the first stone moves at an angle of 30° to its original path with a speed of 1.3 m s^{-1}. Calculate the speed of the second stone after the collision.

18. A cart of mass 250 kg is moving horizontally at 4.0 m s^{-1} when a 50 kg sack of gravel is dropped vertically into it. Calculate the new horizontal speed of the cart.

19. Object C moves at 6.0 m s^{-1} towards object D, which is moving at 2.0 m s^{-1} in the same direction. They collide perfectly elastically. After the collision, object C continues in the same direction at 1.0 m s^{-1}. Calculate the speed of object D after the collision.

20. A raindrop falls vertically through the air. As its speed increases, the air resistance acting on it increases. State and explain the effect of this on the rate of change of momentum of the raindrop.

21. A firework rocket is at the highest point of its trajectory and is momentarily stationary. It explodes into three pieces, A, B and C, which move in the same horizontal plane. Piece A has mass 0.20 kg and moves at 15 m s^{-1} due North. Piece B has mass 0.15 kg and moves at 20 m s^{-1} due East. Piece C moves due South-West (at an angle of 45° South of West) with a speed of 24 m s^{-1}. Calculate the mass of piece C.

22. A spacecraft of total mass 850 kg is travelling in a straight line at a speed of 7500 m s^{-1} in deep space. It ejects a spent rocket booster of mass 250 kg backwards. Following the ejection, the remaining 600 kg of the spacecraft continues in the same direction at an increased speed of 7800 m s^{-1}. (a) Calculate the velocity of the ejected booster. (b) State the direction of motion of the booster relative to a stationary observer in space.

23. Particle P of mass m and speed v collides perfectly inelastically with a stationary particle Q of mass $3m$. They move off together. Show that 75% of the initial kinetic energy is lost in the collision.

24. A rubber ball of mass 0.15 kg strikes a rigid vertical wall at a speed of 12 m s^{-1} at an angle of 90° to the wall. It rebounds perfectly elastically. Calculate the magnitude of the change in momentum of the ball.

25. A small object of mass 0.12 kg is released from rest in a vacuum, where it falls with acceleration $g = 9.81$ m s^{-2}. The same object is then released from rest in air. Its initial acceleration in air is 9.81 m s^{-2}. When its speed is 15 m s^{-1}, its acceleration is 4.2 m s^{-2}. Calculate the magnitude of the air resistance acting on the object at this speed.

Advanced Practice（培优）· 20

1. Sphere A of mass 0.40 kg moves at 5.0 m s^{-1} and collides elastically with sphere B of mass 0.60 kg moving at 2.0 m s^{-1} in the same direction. (a) State what is meant by an elastic collision. (b) Calculate the final velocities of A and B.

2. A car of mass 1200 kg travelling North at 15 m s^{-1} collides with a lorry of mass 3000 kg travelling East at 10 m s^{-1}. They stick together on impact. (a) Calculate the magnitude and direction of their combined velocity immediately after impact. (b) Calculate the loss of kinetic energy during the collision.

3. A skydiver of mass 85 kg falls from a stationary balloon. Air resistance is not negligible. (a) Describe and explain qualitatively the variation of the skydiver's acceleration from the moment of release until terminal velocity is reached. (b) The skydiver opens a parachute, which significantly increases the air resistance. State and explain the immediate effect on the skydiver's acceleration.

4. A stationary nucleus of mass 3.8×10^{-25} kg decays into three particles, A, B, and C. Particle A has mass 6.6×10^{-27} kg and moves at 1.5×10^{7} m s^{-1} along the positive x-axis. Particle B has mass 6.6×10^{-27} kg and moves at 1.5×10^{7} m s^{-1} along the positive y-axis. Calculate the magnitude of the velocity of particle C.

5. Object P of mass m is moving with speed v towards a stationary object Q of mass $3m$. The collision is perfectly elastic. (a) Show that the final speed of P is $0.5v$ and state its direction. (b) Determine the fraction of the initial kinetic energy of P that is transferred to Q.

6. A small metal sphere is released from rest in a tall cylinder of oil. (a) State how the acceleration of the sphere can be deduced from its velocity-time graph. (b) Explain, in terms of the forces acting, why the gradient of the velocity-time graph eventually decreases to zero.

7. Particle A of mass 2.0 kg moves at 4.0 m s^{-1} and collides perfectly elastically with a stationary particle B of mass 2.0 kg. After the collision, particle A moves at an angle of 30° to its original path. Determine the speed of particle A and the speed of particle B after the collision.

8. A railway truck X of mass 4500 kg travels at 2.4 m s^{-1} and collides with a stationary truck Y of mass 2500 kg. After the collision, X continues in the same direction at 1.2 m s^{-1}. (a) Calculate the speed of Y after the collision. (b) Determine whether the collision is elastic or inelastic.

9. A firework rocket is at the highest point of its trajectory, moving horizontally at 12 m s^{-1} East. It explodes into two fragments, M1 and M2, of mass 0.30 kg and 0.50 kg respectively. Immediately after the explosion, M1 moves horizontally at 25 m s^{-1} North. Calculate the magnitude and direction of the velocity of M2 immediately after the explosion.

10. A cyclist is freewheeling down a hill. The total mass is 80 kg. The component of their weight parallel to the slope is 60 N. The drag force D on the cyclist is proportional to the square of their speed v, such that $D = kv^2$, where $k = 0.24$ kg m^{-1}. (a) Determine the terminal velocity of the cyclist on this slope. (b) The cyclist now pedals, providing an additional forward driving force of 40 N. Calculate the new terminal velocity.

11. Three identical blocks, A, B, and C, are on a smooth horizontal surface. Block A moves at 6.0 m s^{-1} towards B, which is stationary. Block C is also stationary, further along the line of motion. The collision between A and B is perfectly elastic. The subsequent collision between B and C is perfectly inelastic. (a) State the speeds of A and B immediately after the first collision. (b) Calculate the speed of the combined B-C system after the second collision. (c) Determine the total percentage of kinetic energy lost in the entire sequence of events.

12. An object is projected vertically upwards from the ground. Air resistance is not negligible. (a) Compare the magnitude of the object's acceleration just after it is projected upwards with its acceleration at the highest point of its trajectory. Explain your reasoning. (b) State and explain whether the time taken to rise to the highest point is greater than, equal to, or less than the time taken to fall back to the ground.

13. Two ice skaters, X and Y, collide and hold on to each other. Skater X has mass 65 kg and is moving East at 4.0 m s^{-1}. Skater Y has mass 55 kg and is moving North at 5.0 m s^{-1}. Calculate the loss in kinetic energy during the collision.

14. A glider of mass m moving at 1.8 m s^{-1} on a linear air track collides elastically with a stationary glider of mass M. After the collision, the glider of mass m rebounds with a speed of 0.6 m s^{-1}. (a) Calculate the speed of the glider of mass M after the collision. (b) Determine the ratio M/m.

15. A ball of mass 0.15 kg falls vertically through a fluid. The drag force D acting on the ball is given by $D = cv$, where v is the velocity and c is a constant. The ball reaches a terminal velocity of 2.5 m s^{-1}. (a) Calculate the constant c, stating its SI base units. (b) Calculate the initial acceleration of the ball when it is dropped from rest. (c) Determine the acceleration of the ball when its velocity is 1.5 m s^{-1}.

16. Two astronauts, P and Q, are floating stationary in space next to each other. Astronaut P has mass 75 kg and astronaut Q has mass 90 kg. P pushes Q away. After the push, Q moves away at a constant speed of 1.2 m s^{-1}. (a) Calculate the speed of P after the push. (b) Calculate the total kinetic energy of the two astronauts after the push. (c) State the source of this kinetic energy.

17. Sphere A of mass 0.20 kg travels at 5.0 m s^{-1} in the positive x-direction. It collides with a stationary sphere B of mass 0.30 kg. After the collision, sphere A moves at 3.0 m s^{-1} at an angle of 53.1° above the positive x-axis. (a) Calculate the y-component of the velocity of sphere B after the collision. (b) Calculate the x-component of the velocity of sphere B after the collision. (c) Determine whether the collision is elastic.

18. Two trolleys, X and Y, are held together on a horizontal track with a compressed spring between them. Trolley X has mass 0.40 kg and trolley Y has mass 0.60 kg. The spring is released, and the trolleys move apart. Trolley X acquires a kinetic energy of 0.45 J. (a) Calculate the speed of trolley X. (b) Calculate the momentum of trolley X. (c) Deduce the kinetic energy of trolley Y.

19. A proton of mass m travels at speed v_0 and collides elastically with a stationary proton. After the collision, the two protons move off at angles θ and ϕ to the original direction of travel, as shown in Fig. 19. (a) Write down an equation relating the initial speed v_0 and final speeds v_1 and v_2 of the protons, based on the conservation of kinetic energy. (b) Use the principle of conservation of momentum and your answer to (a) to show that the angle between the final paths of the protons, $(\theta + \phi)$, is 90°.

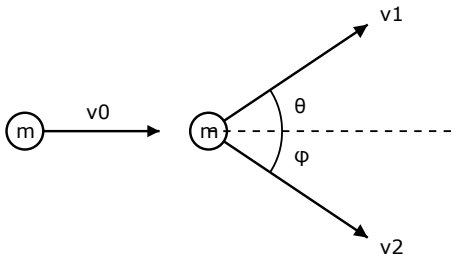

Fig. 19.

20. A raindrop of mass 3.4×10^{-6} kg falls vertically through the air. The air resistance F acting on the drop is given by $F = kv^2$, where v is the speed of the drop and $k = 1.2 \times 10^{-6}$ kg m^{-1}. (a) Calculate the terminal velocity of the raindrop. (b) The raindrop evaporates as it falls, reducing its mass to half of its original value. Assuming k remains constant, calculate the new terminal velocity. (c) Describe qualitatively how the speed of the raindrop changes as it evaporates.

4 Forces, density and pressure（力、密度与压强）

Understanding turning effects of forces, conditions for equilibrium, density, hydrostatic pressure, and Archimedes' principle.

Rules 规则

- The moment of a force is the product of the force and the perpendicular distance from the pivot to the line of action of the force: $M = Fimesd$ (力矩).
- A couple consists of two equal, antiparallel forces whose lines of action do not coincide, producing rotation only. The torque of a couple is the product of one of the forces and the perpendicular distance between their lines of action (力偶与力偶矩).
- For an object to be in equilibrium, the resultant force must be zero and the resultant torque about any point must be zero. The principle of moments states that the sum of clockwise moments equals the sum of anticlockwise moments (平衡条件与力矩原理).
- Density is mass per unit volume: $\rho = \frac{m}{V}$ (密度). Pressure is the normal force applied per unit area: $p = \frac{F}{A}$ (压强).
- Hydrostatic pressure due to a fluid column is $\Delta p = \rho g \Delta h$ (流体静压强).
- Upthrust on an object in a fluid is caused by the pressure difference between its top and bottom surfaces, calculated using Archimedes' principle: $F = \rho g V$, where V is the submerged volume (浮力与阿基米德原理).

Worked Examples 例题

Example 4.1

A uniform wooden beam of length 4.0 m and weight 150 N rests horizontally on two supports, A and B. Support A is at the left end and support B is 1.0 m from the right end. Calculate the upward force exerted by support B.

Solution

Let the left end be the pivot (point A). The centre of gravity of the uniform beam is at its midpoint, 2.0 m from A. The distance from A to B is 4.0 m − 1.0 m = 3.0 m. Applying the principle of moments about A: Sum of clockwise moments = Sum of anticlockwise moments. $150 \times 2.0 = F_{\mathrm{B}} \times 3.0$. $300 = 3.0 F_{\mathrm{B}}$. $F_{\mathrm{B}} = 100$ N.

Example 4.2

An object of weight W is suspended by two light strings. The strings make angles of 30° and 60° to the horizontal. The tension in the string at 30° is 40 N. Determine the weight W of the object.

Solution

The system is in equilibrium, so the forces form a closed vector triangle. The angle between the two strings is $180° - 30° - 60° = 90°$. Resolving horizontally: $T_1\cos(30°) = T_2\cos(60°)$, so $40\cos(30°) = T_2\cos(60°)$, giving $T_2 = 40 \times \frac{\sqrt{3}/2}{1/2} = 40\sqrt{3} \approx 69.3$ N. Resolving vertically: $W = T_1\sin(30°) + T_2\sin(60°) = 40\sin(30°) + 40\sqrt{3}\sin(60°) = 20 + 60 = 80$ N.

Example 4.3

A solid metal cylinder of height 0.15 m and cross-sectional area 0.020 m^2 is completely submerged in water of density 1000 kg m^{-3}. The top surface of the cylinder is 0.20 m below the surface of the water. Calculate the difference in hydrostatic pressure between the bottom and top surfaces of the cylinder, and hence determine the upthrust acting on it. Use $g = 9.81$ m s^{-2}.

Solution

The difference in depth between the bottom and top surfaces is the height of the cylinder, $\Delta h = 0.15$ m. The difference in hydrostatic pressure is $\Delta p = \rho g \Delta h = 1000 \times 9.81 \times 0.15 = 1471.5$ Pa. The upthrust is the force due to this pressure difference: $F = \Delta p \times A = 1471.5 \times 0.020 = 29.43$ N. Alternatively, using Archimedes' principle: $V = A \times h = 0.020 \times 0.15 = 0.0030$ m^3. Upthrust $F = \rho g V = 1000 \times 9.81 \times 0.0030 \approx 29$ N.

考点总览 Key Points

- ☐ 我理解重力可视为集中作用于单一位置，即物体的重心 (centre of gravity)。
- ☐ 我掌握力矩 (moment of a force) 的定义，并能熟练计算力对转轴的转动效应。
- ☐ 我理解力偶 (couple) 只产生纯转动，并能计算力偶矩 (torque of a couple)。
- ☐ 我能陈述并应用力矩原理 (principle of moments) 求解受力平衡问题。
- ☐ 我掌握物体处于平衡状态 (equilibrium) 的充要条件：合力为零且合力矩为零。
- ☐ 我会利用闭合矢量三角形 (vector triangle) 表示和求解共面共点力的平衡问题。
- ☐ 我能定义并运用密度 (density) 公式进行计算（$\rho = \frac{m}{V}$）。
- ☐ 我能定义并运用压强 (pressure) 公式进行计算（$p = \frac{F}{A}$）。
- ☐ 我会推导并运用流体静压强 (hydrostatic pressure) 公式（$\Delta p = \rho g \Delta h$）。
- ☐ 我理解流体中的浮力 (upthrust) 是由上下表面所受的流体静压强差产生的。
- ☐ 我能运用阿基米德原理 (Archimedes' principle) 计算物体受到的浮力（$F = \rho g V$）。

Basic Drill（入门综合）· 50

1. Define the centre of gravity of an object.

2. A force of 45 N acts perpendicularly on a door at a distance of 0.60 m from the hinges. Calculate the moment of the force about the hinges.

3. A couple consists of two parallel forces, each of magnitude 28 N, acting in opposite directions. The perpendicular distance between their lines of action is 0.35 m. Calculate the torque of the couple.

4. State the two conditions required for a rigid body to be in static equilibrium.

5. A uniform horizontal rod of negligible weight is pivoted at its centre. A vertical downward force of 15 N is applied at a distance of 0.40 m to the left of the pivot. Calculate the distance to the right of the pivot where a vertical downward force of 20 N must act to balance the rod.

6. Define the torque of a couple.

7. A solid rectangular block of mass 1.44 kg has dimensions 0.10 m $\times$ 0.080 m $\times$ 0.050 m. Calculate the density of the block.

8. A solid block of mass 6.0 kg rests on a flat horizontal table. The base area of the block in contact with the table is 0.025 m^2. Calculate the pressure exerted by the block on the table.

9. Seawater has a density of 1030 kg m^{-3}. Calculate the hydrostatic pressure at a depth of 15 m below the surface of the sea.

10. A submarine hatch has an area of 0.45 m^2. The difference in pressure between the outside and the inside of the hatch is 2.8×10^5 Pa. Calculate the resultant force exerted on the hatch due to this pressure difference.

11. State what causes the upthrust acting on an object submerged in a fluid.

12. A solid metal sphere of volume 3.5×10^{-4} m^3 is completely submerged in water of density 1000 kg m^{-3}. Calculate the upthrust acting on the sphere.

13. A uniform metre rule of negligible weight is pivoted at the 50 cm mark. A downward force of 3.0 N is applied at the 20 cm mark. Calculate the downward force that must be applied at the 75 cm mark to keep the rule in horizontal equilibrium.

14. An aluminium alloy has a density of 2.7 g cm^{-3}. Convert this density into kg m^{-3}.

15. Three coplanar forces act on a body at a point. The body is in equilibrium. State the geometric shape formed when the vectors representing these three forces are drawn tip-to-tail.

16. A mechanic applies a force of 50 N to the end of a spanner of length 0.30 m. The force acts at an angle of 60° to the length of the spanner. Calculate the moment of the force about the bolt at the other end.

17. A vertical cylinder of height 0.80 m is completely filled with oil of density 850 kg m^{-3}. Calculate the difference in hydrostatic pressure between the bottom and the top of the cylinder.

18. State the principle of moments.

19. A wooden block of total volume 2.0×10^{-3} m^3 floats in water of density 1000 kg m^{-3}. Given that 60% of the volume of the block is submerged, calculate the mass of the block.

20. A point object is held in equilibrium by three coplanar forces: a horizontal force of 12 N to the left, a vertical downward force of 16 N, and a tension force T. Calculate the magnitude of T.

21. A solid cube of side length 0.10 m is completely submerged in water of density 1000 kg m^{-3}. Calculate the magnitude of the upthrust acting on the cube.

22. Define pressure.

23. A uniform horizontal beam of weight 120 N and length 4.0 m is supported by two vertical pillars, one at each end. Calculate the upward force exerted by each pillar on the beam.

24. An iceberg of density 920 kg m^{-3} floats in seawater of density 1025 kg m^{-3}. Determine the percentage of the iceberg's total volume that is submerged beneath the surface.

25. A scuba diver descends from a depth of 5.0 m to a depth of 22.0 m in freshwater of density 1000 kg m^{-3}. Calculate the increase in hydrostatic pressure experienced by the diver.

26. Define the moment of a force about a point.

27. A flat lamina is suspended freely from a pin and hangs in static equilibrium. State the position of the centre of gravity of the lamina relative to the pin.

28. A force of magnitude 45 N acts perpendicularly on a spanner at a distance of 0.24 m from the centre of a nut. Calculate the moment of the force about the centre of the nut.

29. A force of 60 N is applied to a lever at an angle of 35° to the lever. The point of application of the force is 0.75 m from the pivot. Calculate the moment of this force about the pivot.

30. Define a couple.

31. A mechanic turns a valve wheel by applying a couple consisting of two equal and opposite forces, each of magnitude 35 N. The torque produced is 4.2 N m. Calculate the perpendicular distance between the lines of action of the two forces.

32. Two parallel forces, each of magnitude 18 N, act in opposite directions on the rim of a steering wheel of diameter 0.38 m. Calculate the torque of the couple.

33. A light horizontal rod of length 0.80 m is pivoted at one end. A vertical upward force F is applied at the free end to support a downward load of 60 N placed at a distance of 0.20 m from the pivot. Calculate the magnitude of F required to maintain equilibrium.

34. State the two conditions necessary for a system of coplanar forces to be in equilibrium.

35. A uniform metre ruler is pivoted at the 50.0 cm mark. A weight of 1.8 N is suspended from the 15.0 cm mark. Calculate the weight of the object that must be suspended from the 75.0 cm mark to keep the ruler in horizontal equilibrium.

36. A uniform beam of weight 240 N and length 3.0 m rests horizontally on two vertical supports, one at each end X and Y. A load of 360 N is placed at a distance of 1.0 m from support X. Calculate the upward force exerted by support X on the beam.

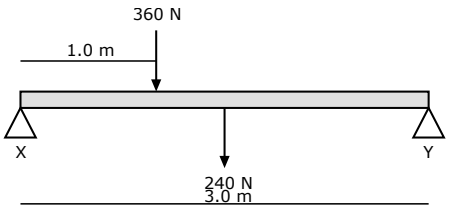

Fig. 36. Uniform beam supported at ends X and Y with applied load.

37. A small mass of weight 12 N is suspended from a light string and held in equilibrium by a horizontal force F, so that the string makes an angle of 30° with the vertical. Calculate the magnitude of F.

38. Explain why three coplanar forces acting on an object in equilibrium can be represented by the sides of a closed vector triangle.

39. Define density.

40. A solid metal cylinder has a radius of 0.020 m, a height of 0.15 m, and a mass of 1.6 kg. Calculate the density of the metal.

41. Convert a density of 2.7 g cm^{-3} into kg m^{-3}.

42. Define pressure.

43. A solid box of mass 45 kg rests on a horizontal floor. The base of the box in contact with the floor has dimensions 0.60 m $\times$ 0.50 m. Calculate the pressure exerted by the box on the floor.

44. Calculate the increase in hydrostatic pressure at a depth of 25 m below the surface of water of density 1.0×10^3 kg m^{-3}.

45. The hydrostatic pressure at a depth h in a liquid of density 880 kg m^{-3} is 5.2×10^4 Pa. Calculate h.

46. Explain why a fluid exerts an upward force (upthrust) on a fully submerged object.

47. A solid object is completely immersed in paraffin of density 800 kg m^{-3} and experiences an upthrust of 3.6 N. Calculate the volume of the object.

48. A block of wood of density 720 kg m^{-3} floats in an oil with 80% of its volume submerged beneath the surface. Calculate the density of the oil.

49. A solid object has a weight of 8.2 N in air and an apparent weight of 5.7 N when completely immersed in water. Calculate the upthrust acting on the object.

50. A vertical cylinder of liquid has a base area A, height h, and uniform density ho. Starting from the weight of the liquid column, show that the hydrostatic pressure Δp at the base is given by $\Delta p = \rho g h$.

Practice（综合练习）· 25

1. A traffic light of mass 22 kg is suspended by two light cables, A and B. The traffic light is in equilibrium. Cable A makes an angle of 30° to the horizontal and cable B makes an angle of 60° to the horizontal.
   (a) State the two conditions for the traffic light to be in equilibrium.
   (b) Calculate the tension in cable A.
   (c) Calculate the tension in cable B.

2. A flat-bottomed boat has a rectangular base of length 12.0 m and width 4.5 m. The boat floats in seawater of density 1025 kg m^{-3}. The total mass of the boat is 2.80×10^4 kg.
   (a) Calculate the upthrust acting on the boat.
   (b) Calculate the depth of the boat's base below the surface of the seawater.
   (c) The boat sails into a freshwater river of density 1000 kg m^{-3}. Determine the distance by which the boat sinks further into the water.

3. A small object of weight 5.0 N is suspended by a light string. A horizontal force H is applied to the object so that the string makes an angle of 30° to the vertical. The object is in equilibrium.
   (a) State the two conditions for an object to be in equilibrium.
   (b) Calculate the horizontal force H.

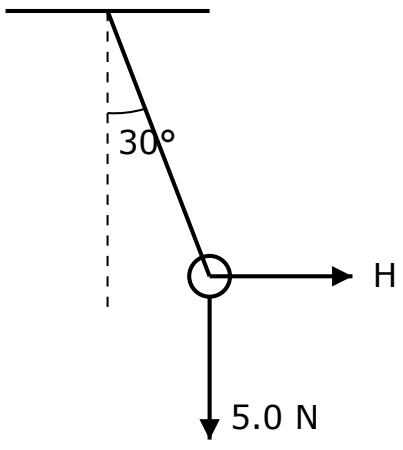

Fig. 3. ure for Question 53

4. A solid rectangular block has dimensions 0.20 m × 0.15 m × 0.10 m and a mass of 12 kg.
   (a) Calculate the density of the block.
   (b) The block rests on a flat horizontal surface. Calculate the maximum pressure the block can exert on the surface.

5. (a) Use the definitions of pressure and density to derive the equation $\Delta p = \rho g \Delta h$ for the hydrostatic pressure in a liquid.
   (b) A diver operates at a depth of 25 m in sea water of density 1030 kg m^{-3}. Calculate the pressure exerted by the water on the diver.

6. A solid metal sphere of radius 0.050 m is completely submerged in water of density 1000 kg m^{-3}.
   (a) Explain, in terms of pressure difference, why an upthrust acts on the sphere.
   (b) Calculate the upthrust acting on the sphere.

7. A uniform horizontal trapdoor of weight 85 N and length 1.2 m is hinged at one end. It is held open in a horizontal position by a light wire attached to the other end. The wire makes an angle of 45° with the trapdoor.
   (a) State the principle of moments.
   (b) Calculate the tension in the wire.

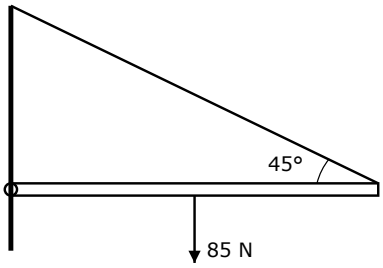

Fig. 7. ure for Question 57

8. A picture of weight W is hung from a smooth nail by a single light string. The ends of the string are attached to two points on the top edge of the picture. The string passes over the nail. The tension in the string is 15 N and the string makes an angle of 40° to the horizontal on both sides of the nail.
   Determine the weight W of the picture.

9. A non-uniform beam AB of length 3.0 m is balanced horizontally on two supports at C and D. The support C is 0.50 m from A and the support D is 1.0 m from B. The upward force exerted by support C is 240 N and the upward force exerted by support D is 160 N.
   (a) Calculate the weight of the beam.
   (b) Determine the distance of the centre of gravity of the beam from end A.

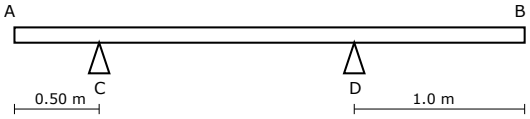

Fig. 9. ure for Question 59

10. A U-tube manometer contains two immiscible liquids, liquid X and liquid Y. The liquids are in equilibrium. The column of liquid X has a height of 18.0 cm and density 850 kg m^{-3}. It balances a column of liquid Y of height 12.5 cm.
    Calculate the density of liquid Y.

11. A stone of mass 2.4 kg and density 2800 kg m^{-3} is suspended by a string and completely immersed in a liquid of density 1200 kg m^{-3}. The stone does not touch the bottom of the container.
    (a) Calculate the volume of the stone.
    (b) Calculate the tension in the string.

12. A uniform circular disc of diameter 0.80 m is pivoted at its centre. Two forces, each of magnitude 45 N, act tangentially on opposite edges of the disc to form a couple. A third force F acts tangentially on the edge of the disc at a distance of 0.25 m from the pivot to keep the disc in equilibrium.
    Calculate the magnitude of force F.

13. An alloy is made by mixing 4.0×10^{-3} m^3 of metal P with 6.0×10^{-3} m^3 of metal Q. Metal P has a density of 7200 kg m^{-3} and metal Q has a density of 8900 kg m^{-3}. The total volume of the alloy is equal to the sum of the volumes of the two metals.
    Calculate the density of the alloy.

14. Water is supplied to a building. The water pressure at ground level is 4.5×10^5 Pa. The density of water is 1000 kg m^{-3}.
Calculate the water pressure at a tap located 15 m above ground level.

15. A tower crane consists of a horizontal beam supported by a vertical pillar. The horizontal beam has a uniform weight of 2.5×10^4 N and a length of 30 m. The pillar is attached to the beam at a distance of 10 m from the left end. A counterweight of weight W is placed at the left end. A load of 1.2×10^4 N is suspended from the right end.
Calculate the weight of the counterweight W required so that the beam is in horizontal equilibrium.

16. A submarine is submerged in sea water of density 1030 kg m^{-3}. The pressure inside the submarine is equal to atmospheric pressure, 1.01×10^5 Pa. The submarine has a circular escape hatch of diameter 0.60 m located at a depth of 45 m.
(a) Calculate the absolute hydrostatic pressure on the outside of the hatch.
(b) Calculate the resultant inward force on the hatch.

17. A rectangular block of wood of density 650 kg m^{-3} floats in water of density 1000 kg m^{-3}. The block has a total height of 0.12 m.
Calculate the depth of the block that is submerged in the water.

18. A uniform diving board of length 4.0 m and mass 45 kg is supported by two pillars. Pillar A is at the left end of the board and pillar B is 1.2 m from the left end. A diver of mass 65 kg stands at the right end of the board.
Calculate the magnitude of the vertical force exerted by pillar A on the board.

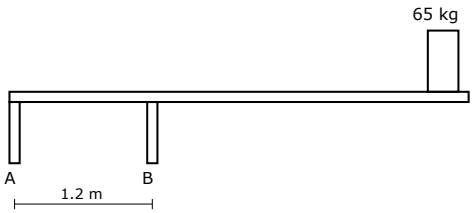

Fig. 18. ure for Question 68

19. A smooth inclined plane makes an angle of 25° with the horizontal. A block of weight 35 N is held in equilibrium on the plane by a light string that is parallel to the plane.
    (a) State the forces acting on the block.
    (b) Calculate the tension in the string.

20. A hot air balloon has a constant volume of 2500 m^3. The density of the surrounding cool air is 1.22 kg m^{-3}. The total mass of the balloon fabric, basket, and passengers is 450 kg. The balloon is floating in equilibrium in the air.
    (a) Calculate the upthrust acting on the balloon.
    (b) Calculate the maximum density of the hot air inside the balloon for it to remain in equilibrium.

21. A car driver applies a couple to a steering wheel of diameter 0.38 m. The driver applies a force of 25 N to each side of the wheel.
    (a) State what is meant by a couple.
    (b) Calculate the torque produced by the driver.

22. A woman of mass 60 kg stands on one heel of area 1.2×10^{-4} m^2. An elephant of mass 4500 kg stands on four feet, each of area 0.15 m^2.
    (a) Calculate the pressure exerted on the ground by the woman's heel.
    (b) Calculate the pressure exerted on the ground by the elephant.

23. A small sphere of mass 0.15 kg hangs from a fixed point by a light inextensible string. A horizontal breeze exerts a constant force F on the sphere. The sphere is in equilibrium when the string is at an angle of 15° to the vertical.
    Determine the magnitude of the force F.

24. A student uses a uniform metre rule to find the mass of a rock. The rule is pivoted at the 30.0 cm mark. The rock, of mass 0.24 kg, is suspended from the 10.0 cm mark. The rule is in horizontal equilibrium.
    Calculate the mass of the metre rule.

25. A pressure gauge is lowered into a deep lake. The density of the lake water is 1000 kg m^{-3}. The atmospheric pressure at the surface is 1.01×10^5 Pa. The pressure gauge reads 8.50×10^5 Pa.
    Calculate the depth of the pressure gauge below the surface of the lake.

Advanced Practice（培优）· 20

1. A uniform beam of length 4.0 m and mass 25 kg rests horizontally on two supports, A and B. Support A is at the left end, and support B is 1.0 m from the right end. A person of mass 65 kg stands on the beam at a distance of 1.5 m from the left end. (a) Define the moment of a force. (b) Calculate the reaction force at support B. (c) Calculate the reaction force at support A.

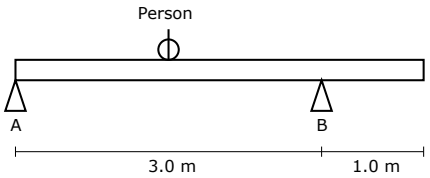

Fig. 1. Diagram showing the beam on two supports.

2. A picture of weight 45 N is suspended by a light string that passes over a smooth nail. The string forms an angle of 40° to the horizontal on both sides of the nail. (a) State the two conditions for a system to be in equilibrium. (b) Calculate the tension in the string. (c) The string is adjusted so that the angle to the horizontal decreases. State and explain the effect on the tension in the string.

3. A mechanic uses a cross wrench to tighten a wheel nut. The mechanic applies two equal and opposite forces of 85 N at the ends of the wrench, which are 0.40 m apart. (a) State what is meant by a couple. (b) Calculate the torque applied to the nut. (c) The mechanic now uses a single-ended wrench of length 0.30 m and applies a single force at the end perpendicular to the wrench. Calculate the force required to produce the same torque as in (b).

4. A U-tube manometer contains water of density 1000 kg m^{-3}. A liquid X of unknown density is poured into one arm. The liquids do not mix. When in equilibrium, the column of liquid X has a height of 15.0 cm and the water level in the other arm is 12.0 cm above the water level in the arm containing liquid X. (a) Derive the equation $\Delta p = \rho g \Delta h$ from the definitions of pressure and density. (b) Calculate the density of liquid X.

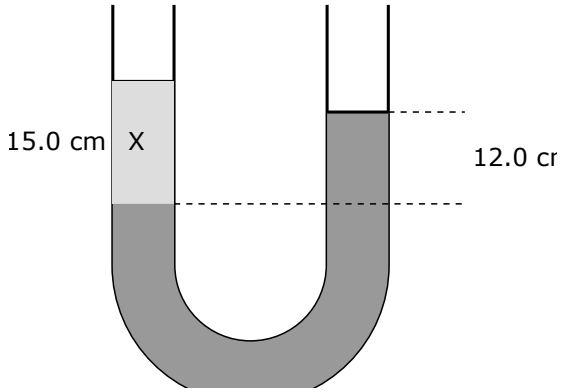

Fig. 4. U-tube manometer with liquid X and water.

5. A solid cylinder of radius 4.5 cm and length 20.0 cm floats vertically in water of density 1000 kg m^{-3}. The density of the material of the cylinder is 720 kg m^{-3}. (a) State Archimedes' principle. (b) Calculate the weight of the cylinder. (c) Calculate the depth of the cylinder that is submerged in the water.

6. A spherical underwater sensor buoy is tethered to the seabed by a straight light cable. The buoy is a uniform solid sphere of radius 0.400 m and mass 120 kg. It is completely submerged in seawater of density 1025 kg m^{-3}. A horizontal ocean current exerts a constant force of 850 N on the buoy. The buoy is in equilibrium.
   (a) Calculate the upthrust acting on the buoy.
   (b) Determine the magnitude of the tension in the cable.
   (c) Calculate the angle the cable makes with the vertical.

7. A non-uniform crane arm AB of length 6.0 m and mass 120 kg is hinged to a vertical wall at end A. The arm is held in a horizontal position by a light cable attached to end B. The cable makes an angle of 25° with the horizontal arm. The tension in the cable is 1500 N.
   (a) Define the centre of gravity of an object.
   (b) Calculate the distance of the centre of gravity of the arm from the hinge at A.
   (c) A load is now suspended from end B. The maximum safe tension in the cable is 2500 N. Calculate the maximum mass of the load that can be safely suspended.

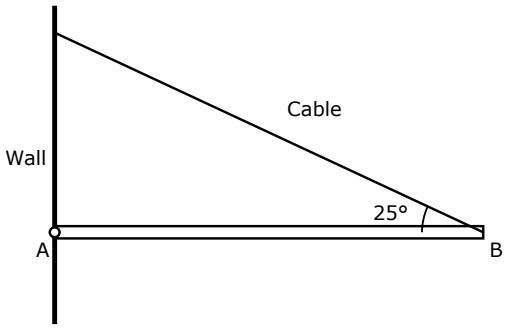

Fig. 7. A diagram showing the crane arm AB hinged at the wall and supported by a cable.

8. A submarine has a circular observation window of diameter 0.30 m. The submarine is submerged in seawater of density 1030 kg m^{-3}. The centre of the window is at a depth of 450 m below the surface of the sea. The pressure inside the submarine is 1.01×10^5 Pa. The atmospheric pressure at the sea surface is also 1.01×10^5 Pa. (a) Define pressure. (b) Calculate the hydrostatic pressure on the window due to the seawater. (c) Calculate the resultant force acting on the window.

9. A block of mass 8.0 kg rests on a smooth inclined plane that makes an angle of 30° with the horizontal. The block is held in equilibrium by a horizontal force F. (a) State the names of the three forces acting on the block. (b) Calculate the magnitude of the horizontal force F. (c) Calculate the normal reaction force exerted by the plane on the block.

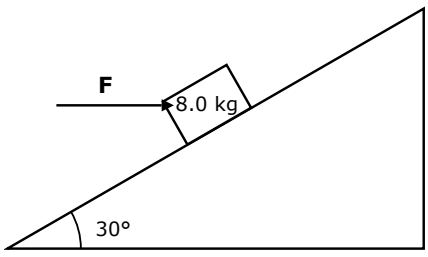

Fig. 9. A block of mass 8.0 kg on a smooth inclined plane at 30° to the horizontal, held in equilibrium by a horizontal force F.

10. A solid uniform cylinder of length 0.240 m and cross-sectional area 4.50×10^{-3} m^2 is in vertical equilibrium across the horizontal boundary between two immiscible liquids. The upper liquid is oil of density 800 kg m^{-3} and the lower liquid is water of density 1000 kg m^{-3}. The cylinder has 0.150 m of its length in the water and 0.090 m of its length in the oil. The top surface of the cylinder is horizontal and is at a depth of 0.100 m below the free surface of the oil.
(a) Calculate the hydrostatic pressure at the top surface of the cylinder.
(b) Calculate the hydrostatic pressure at the bottom surface of the cylinder.
(c) Calculate the upthrust acting on the cylinder.
(d) Determine the density of the material of the cylinder.

11. A uniform rectangular sign of weight 120 N and width 0.80 m is pivoted at its top left corner. A horizontal force F is applied at the bottom right corner to keep the sign in equilibrium such that its top edge is horizontal. The height of the sign is 0.60 m. (a) State the principle of moments. (b) Calculate the magnitude of force F. (c) Calculate the magnitude of the reaction force at the pivot.

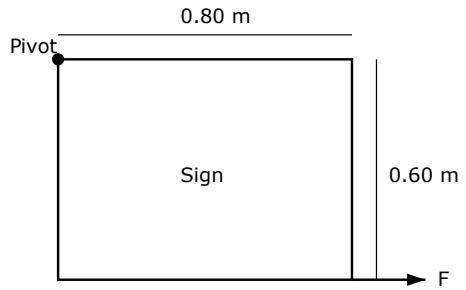

Fig. 11. Rectangular sign pivoted at top left.

12. A uniform rigid lever AB of length 0.600 m and weight 15.0 N is freely pivoted at end A. A circular safety valve of diameter 0.040 m and negligible weight is attached to the lever at a distance of 0.150 m from the pivot A. The valve covers a circular hole of the same diameter in the flat top of a boiler. A counterweight of mass M is fixed to the lever at a distance of 0.500 m from the pivot A. The lever is horizontal and the valve is closed.
The atmospheric pressure outside the boiler is 1.01×10^5 Pa. The valve is designed to open when the absolute pressure of the steam inside the boiler reaches 3.50×10^5 Pa.
(a) Calculate the resultant upward force exerted on the valve by the steam and the atmosphere at the instant the valve opens.
(b) Calculate the required mass M of the counterweight.
(c) The counterweight is moved to end B of the lever. Explain the effect this has on the pressure required to open the valve.

13. A rigid straight beam AB of length 3.0 m rests on two supports, one at A and one at B. The beam has a weight of 150 N. The centre of gravity of the beam is 1.2 m from A. A load of weight W is placed on the beam at a distance of 2.5 m from A. The reaction force at support B is twice the reaction force at support A. (a) State the relationship between the reaction forces and the weights for the beam to be in vertical equilibrium. (b) Determine the value of W.

14. Three coplanar forces of magnitudes 40 N, 60 N and F act on a point object. The object is in equilibrium. The angle between the 40 N force and the 60 N force is 120°. (a) State the conditions for this point object to be in equilibrium. (b) Determine the magnitude of the force F. (c) Determine the angle between the force F and the 60 N force.

15. A uniform solid cube of side length 0.15 m is suspended from a spring balance. Half of the cube's volume is submerged in a liquid of density 1200 kg m^{-3}. The reading on the spring balance is 18 N. (a) Calculate the upthrust acting on the cube. (b) Calculate the density of the material of the cube.

16. A uniform rod of weight W and length L is hinged at its lower end to a vertical wall. It is held at an angle of 60° to the vertical by a horizontal string attached to its upper end. (a) By taking moments about the hinge, show that the tension T in the string is given by $T = \frac{W}{2}\tan 60°$. (b) The weight of the rod is 45 N. Calculate the magnitude of the reaction force at the hinge. (c) Determine the angle the reaction force at the hinge makes with the horizontal.

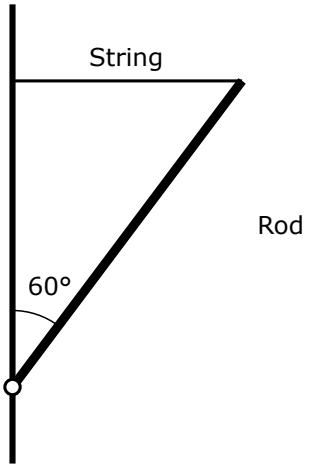

Fig. 16. Rod hinged at a wall and supported by a horizontal string.

17. A solid cylinder of cross-sectional area A and length h is completely submerged in a fluid of constant density ρ. The top surface of the cylinder is horizontal and is at a depth x below the fluid surface. (a) Write down an expression for the hydrostatic pressure p_1 at the top surface and the hydrostatic pressure p_2 at the bottom surface of the cylinder. (b) Use your expressions in (a) to show that the upthrust F acting on the cylinder is given by $F = \rho g V$, where V is the volume of the cylinder. (c) State Archimedes' principle.

18. A hollow sphere has an external radius of 12.0 cm and an internal radius of 11.0 cm. The material of the sphere has a density of 7800 kg m^{-3}. The sphere is placed in a large tank of water of density 1000 kg m^{-3}. (a) Calculate the weight of the sphere. (b) Determine whether the sphere will float or sink in the water. Show your working.

19. A uniform plank of length 5.0 m and mass 40 kg overhangs a horizontal roof. The plank is perpendicular to the edge of the roof, with 1.8 m of its length overhanging. A person of mass 75 kg walks along the overhanging part of the plank. (a) Calculate the maximum distance the person can walk past the edge of the roof before the plank starts to tip. (b) The person holds a 15 kg box. Calculate the new maximum distance the person can walk past the edge.

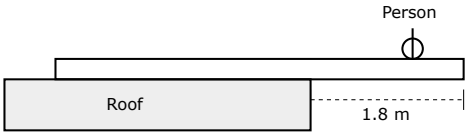

Fig. 19. Uniform plank overhanging a roof.

20. A trapdoor AB is uniform, has a length of 1.2 m and a weight of 250 N. It is hinged at A and is held open at an angle of 30° to the horizontal by a vertical string attached at B. (a) Calculate the tension in the string. (b) The string is now moved so that it is attached to B but acts perpendicular to the trapdoor. Calculate the new tension in the string.

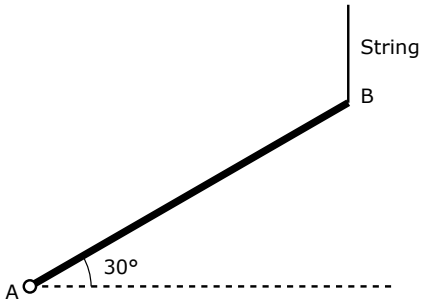

Fig. 20. Trapdoor held open by a vertical string.

5 Work, energy and power（功、能与功率）

Principles of work, energy conservation, power, and the calculation of kinetic and gravitational potential energy.

Rules 规则

- Work done is the product of force and displacement in the direction of the force: $W = Fs$ (功是力与在力的方向上的位移的乘积).
- The principle of conservation of energy states that energy cannot be created or destroyed, only transferred from one form to another (能量守恒定律指出能量既不能凭空产生也不能凭空消失，只能从一种形式转化为另一种形式).
- Efficiency is the ratio of useful energy output to total energy input, often expressed as a percentage (效率是系统输出的有用能量与输入的总能量之比).
- Power is the rate of work done or energy transferred: $P = \frac{W}{t}$ (功率是单位时间内所做的功).
- For an object moving at constant velocity v under a constant driving force F, power can be calculated using $P = Fv$ (对于在恒定驱动力下以恒定速度运动的物体，功率可表示为 $P = Fv$).
- Gravitational potential energy change in a uniform field is $\Delta E_P = mg\Delta h$ (重力势能的变化量), and kinetic energy is $E_K = \frac{1}{2}mv^2$ (动能).

Worked Examples 例题

Example 5.1

A car of mass 1200 kg travels at a constant velocity of 25 m s^{-1} against a total resistive force of 600 N. Calculate the power output of the engine.

Solution

Since the car travels at a constant velocity, the driving force F provided by the engine must equal the total resistive force. Therefore, $F = 600$ N. The power output can be calculated using the formula $P = Fv$. Substituting the values gives $P = 600 \times 25 = 15000$ W. The power output of the engine is 15 kW.

Example 5.2

An electric motor is used to lift a mass of 50 kg vertically upwards through a height of 12 m in 20 s. The constant electrical power input to the motor is 400 W. Calculate the efficiency of the motor.

Solution

First, calculate the useful work done, which is the gain in gravitational potential energy: $\Delta E_P = mg\Delta h = 50 \times 9.81 \times 12 = 5886$ J. The useful power output is $P_{\text{out}} = \frac{W}{t} = \frac{5886}{20} = 294.3$ W. The efficiency is the ratio of useful power output to total power input: Efficiency $= \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{294.3}{400} \times 100\% = 73.6\%$. The efficiency of the motor is 74%.

Example 5.3

A ball of mass 0.20 kg is dropped from rest from a height of 5.0 m. Ignoring air resistance, calculate its kinetic energy and speed just before it hits the ground.

Solution

By the principle of conservation of energy, the loss in gravitational potential energy equals the gain in kinetic energy. Loss in GPE is $\Delta E_P = mg\Delta h = 0.20 \times 9.81 \times 5.0 = 9.81$ J. Therefore, the kinetic energy E_K just before hitting the ground is 9.81 J. Using the formula $E_K = \frac{1}{2}mv^2$, we have $9.81 = \frac{1}{2} \times 0.20 \times v^2$. Solving for v^2 gives $v^2 = 98.1$, so $v = \sqrt{98.1} = 9.9$ m s^{-1}.

Example 5.4

A cyclist and bicycle have a combined mass of 80 kg. They are travelling at 8.0 m s^{-1} when the cyclist applies the brakes, coming to a stop over a distance of 12 m. Calculate the average braking force.

Solution

The initial kinetic energy of the cyclist and bicycle is $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 80 \times (8.0)^2 = 2560$ J. The final kinetic energy is 0 J. The work done W by the braking force equals the change in kinetic energy, so $W = 2560$ J. Using the formula for work done $W = Fs$, the average braking force is $F = \frac{W}{s} = \frac{2560}{12} = 213.3$ N. The average braking force is 210 N (to 2 significant figures).

考点总览 Key Points

- ☐ 我理解做功 (work) 的概念，并能根据沿力方向的位移计算功（$W = Fs$）
- ☐ 我掌握能量守恒定律 (principle of conservation of energy) 并能分析能量转化
- ☐ 我理解系统效率 (efficiency) 的定义，并能利用有用输出与总输入的比值解决问题
- ☐ 我能理解功率 (power) 为单位时间做的功，并熟练运用公式 $P = \frac{W}{t}$ 求解
- ☐ 我会推导功率公式 $P = Fv$，并能结合力与速度解决实际功率问题
- ☐ 我会利用 $W = Fs$ 推导匀强引力场中的重力势能 (gravitational potential energy) 公式 $\Delta E_p = mg\Delta h$
- ☐ 我能熟练运用公式 $\Delta E_p = mg\Delta h$ 计算重力势能的变化量
- ☐ 我会结合运动学公式 (equations of motion) 推导动能 (kinetic energy) 公式 $E_k = \frac{1}{2}mv^2$
- ☐ 我能熟练运用动能公式 $E_k = \frac{1}{2}mv^2$ 进行计算与能量分析

Basic Drill（入门综合）· 50

1. Define power.

2. A constant horizontal force of 45 N is applied to push a box across a smooth floor through a distance of 8.0 m. Calculate the work done on the box.

3. A car of mass 1200 kg travels along a straight road at a constant speed of 25 m s^{-1}. Calculate the kinetic energy of the car.

4. A package of mass 15 kg is raised vertically through a height of 4.0 m in a uniform gravitational field. Calculate the increase in gravitational potential energy of the package.

5. A motor performs 18 kJ of work in a time interval of 45 s. Calculate the power output of the motor.

6. An electric machine is supplied with 800 J of energy. It transfers 520 J to useful work. Calculate the efficiency of the machine.

7. A worker pulls a crate along a horizontal floor by exerting a force of 160 N at an angle of 30° above the horizontal. Calculate the work done by the worker when the crate moves a distance of 12 m.

8. A car moves at a constant velocity of 22 m s^{-1} along a level road against a total resistive force of 650 N. Calculate the power developed by the car's engine.

9. State the principle of conservation of energy.

10. An object of mass 0.50 kg increases its speed from 6.0 m s^{-1} to 10.0 m s^{-1}. Calculate the increase in kinetic energy of the object.

11. A ball of mass 0.20 kg is released from rest at a height of 5.0 m above the ground. Air resistance is negligible. Calculate the speed of the ball just before it hits the ground.

12. A crane lifts a load of mass 750 kg vertically upwards at a constant speed of 0.80 m s^{-1}. Calculate the useful power output of the crane.

13. A pump has an efficiency of 70%. It is used to perform 4.2 kJ of useful work. Calculate the total energy input required by the pump.

14. An electron of mass 9.11×10^{-31} kg has a kinetic energy of 4.8×10^{-17} J. Calculate the speed of the electron.

15. Show that the power P delivered by a constant force F acting on an object moving at a constant velocity v in the direction of the force is given by $P = Fv$, starting from $P = \frac{W}{t}$.

16. A trolley of mass 2.0 kg moves at an initial speed of 4.0 m s^{-1} along a horizontal surface. A constant frictional force of 5.0 N acts on the trolley until it comes to rest. Calculate the distance travelled by the trolley while stopping.

17. A force acts on an object in the direction of its motion. The force is 20 N for the first 4.0 m and then decreases linearly to 0 N at a displacement of 6.0 m, as shown. Determine the total work done by the force.

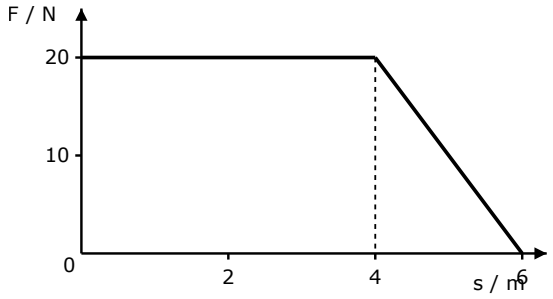

Fig. 17. Graph of force F against displacement s

18. A skier of mass 65 kg travels a distance of 50 m down a slope inclined at 25° to the horizontal. Calculate the decrease in gravitational potential energy of the skier.

19. A power generator receives an energy input at a rate of 2.5 kW and delivers useful electrical power at 2.1 kW. Calculate the rate at which energy is wasted by the generator.

20. Using the equation of motion $v^2 = u^2 + 2as$ and the relation $W = Fs$, derive the formula $E_K = \frac{1}{2}mv^2$ for an object of mass m accelerated from rest ($u = 0$) by a constant resultant force F.

21. A cyclist and bicycle of total mass 80 kg travel at a constant speed of 9.0 m s^{-1} up an incline that rises 1.0 m vertically for every 20 m along the road. Resistive forces are negligible. Calculate the power required from the cyclist.

22. A vehicle of mass 1500 kg moving at 18 m s^{-1} is brought to rest by its brakes on a level road. Calculate the total thermal energy generated in the braking system.

23. A motorboat engine develops a power of 15 kW while driving the boat through water at a steady speed of 6.0 m s^{-1}. Calculate the total drag force acting on the boat.

24. Show that the change in gravitational potential energy ΔE_P of a mass m lifted vertically by a height Δh in a uniform gravitational field of strength g is given by $\Delta E_P = mg\Delta h$, using $W = Fs$.

25. An electric hoist with an efficiency of 85% raises a mass of 240 kg through a vertical height of 15 m in 20 s. Calculate the electrical power input to the hoist.

26. Define power.

27. A box is pulled across a horizontal floor by a horizontal force of 45 N through a distance of 8.0 m. Calculate the work done on the box.

28. A sledge is pulled across a horizontal surface by a rope. The tension in the rope is 50 N at an angle of 60° above the horizontal. Calculate the work done by the tension when the sledge moves a horizontal distance of 12 m.

29. An electron of mass 9.11×10^{-31} kg moves with a speed of 4.0×10^{6} m s^{-1}. Calculate the kinetic energy of the electron.

30. A diver of mass 65 kg stands on a diving board at a height of 10.0 m above the water surface. Calculate the gravitational potential energy of the diver relative to the water surface.

31. A cyclist travels along a horizontal road at a constant speed of 8.0 m s^{-1} against a total resistive force of 35 N. Calculate the power output of the cyclist.

32. An electric motor is supplied with 1.2 kJ of electrical energy and delivers 900 J of useful mechanical work. Calculate the efficiency of the motor.

33. A football of mass 0.42 kg has a kinetic energy of 84 J. Calculate the speed of the football.

34. A car of mass 1200 kg accelerates from rest to a speed of 15 m s^{-1} along a horizontal road. Calculate the work done by the resultant force on the car.

35. A stone is dropped from rest from the top of a cliff of height 45 m. Assuming air resistance is negligible, calculate the speed of the stone immediately before it hits the ground.

36. An electric winch rated at 1.5 kW operates for a time of 4.0 minutes. Calculate the total energy supplied to the winch.

37. A water pump operates with an efficiency of 60%. The useful power output of the pump is 450 W. Calculate the total power input to the pump.

38. A crate of mass 25 kg is pushed up a smooth ramp of length 6.0 m inclined at an angle of 30° to the horizontal. Calculate the work done against gravity in moving the crate to the top of the ramp.

39. State the equation of motion for uniform acceleration that is combined with $W = Fs$ and $F = ma$ to derive the formula for kinetic energy $E_{\mathrm{K}} = \frac{1}{2}mv^2$.

40. A car travelling at 24 m s^{-1} has a kinetic energy of 2.16×10^5 J. A constant braking force of 4800 N brings the car to rest. Calculate the distance travelled by the car while braking.

41. An aeroplane of mass 8.0×10^4 kg climbs from an altitude of 2.0 km to 11.0 km. Assuming the gravitational field is uniform, calculate the gain in gravitational potential energy of the aeroplane.

42. Object P has mass $2m$ and speed v. Object Q has mass m and speed $3v$. Determine the ratio $\frac{\text{kinetic energy of Q}}{\text{kinetic energy of P}}$.

43. A person of mass 70 kg runs up a flight of stairs of vertical height 6.0 m in a time of 5.0 s. Calculate the average power developed by the person against gravity.

44. A car moves at a constant maximum speed of 40 m s^{-1} on a level road. The power output of the engine is 60 kW. Calculate the total resistive force acting on the car.

45. A machine is supplied with 5.0 kJ of energy. During operation, 1.8 kJ of energy is wasted as thermal energy to the surroundings. Calculate the efficiency of the machine.

46. A force acting on an object increases linearly from 0 N to 50 N over a displacement of 4.0 m in the direction of the force. Calculate the work done by the force.

47. A ball of mass 0.15 kg is thrown vertically upwards with an initial kinetic energy of 30 J. Assuming air resistance is negligible, calculate the maximum height reached by the ball above its release point.

48. A gardener pushes a lawnmower a distance of 15 m along level ground, doing 720 J of work. The force is applied along the handle, which is inclined at an angle of 35° below the horizontal. Calculate the magnitude of the force applied along the handle.

49. In a hydroelectric scheme, water falls through a vertical height of 25 m at a rate of 400 kg s^{-1}. Calculate the rate at which gravitational potential energy is lost by the falling water.

50. A trolley of mass 2.0 kg is released from rest at the top of a frictionless curved track of vertical height 1.8 m. Calculate the speed of the trolley at the bottom of the track.

Practice（综合练习）· 25

1. (a) Define work done. (b) A constant force of 45 N is applied to a box at an angle of 30° to the horizontal. The box moves a distance of 8.0 m along a horizontal surface. Calculate the work done by the force.

2. (a) State the principle of conservation of energy. (b) A ball of mass 0.15 kg is thrown vertically upwards with an initial speed of 12 m s^{-1}. Air resistance is negligible. (i) Calculate the initial kinetic energy of the ball. (ii) Use the principle of conservation of energy to determine the maximum height reached by the ball.

3. (a) Use the equation for work done to derive the formula $\Delta E_P = mg\Delta h$ for the change in gravitational potential energy of a mass m moved vertically through a height Δh in a uniform gravitational field. (b) A crane lifts a concrete block of mass 450 kg vertically through a height of 15 m at a constant speed. Calculate the increase in gravitational potential energy of the block.

4. (a) Use the equations of motion to derive the formula $E_K = \frac{1}{2}mv^2$ for the kinetic energy of an object of mass m moving with speed v. (b) A car of mass 1200 kg accelerates from rest to a speed of 25 m s^{-1}. Calculate the increase in kinetic energy of the car.

5. (a) Define power. (b) A water pump lifts 250 kg of water per minute through a vertical height of 8.5 m. The water is ejected with a speed of 4.0 m s^{-1}. (i) Calculate the change in gravitational potential energy of the water per minute. (ii) Calculate the kinetic energy of the water ejected per minute. (iii) Determine the useful power output of the pump.

6. A car of mass 1500 kg travels at a constant speed of 32 m s^{-1} along a straight, horizontal road. The total resistive force acting on the car is 650 N. (a) Derive the formula $P = Fv$ relating power P, force F and speed v. (b) Calculate the power required from the car's engine to maintain this constant speed.

7. An electric motor is used to lift a load of mass 75 kg vertically upwards. The motor is supplied with an electrical power of 1.2 kW and has an efficiency of 65%. (a) Calculate the useful power output of the motor. (b) Calculate the constant speed at which the load is lifted.

8. A cyclist and her bicycle have a combined mass of 82 kg. She is riding up a straight hill inclined at an angle of 4.0° to the horizontal at a constant speed of 5.5 m s^{-1}. A constant resistive force of 45 N acts against her motion. (a) Calculate the rate at which the cyclist is gaining gravitational potential energy. (b) Calculate the rate at which the cyclist is doing work against the resistive force. (c) Determine the total useful power output of the cyclist.

9. An archer pulls a bowstring back a distance of 0.65 m by applying an average force of 140 N. The arrow to be fired has a mass of 0.024 kg. (a) Calculate the work done in pulling the bowstring. (b) When released, 80% of the energy stored in the bow is transferred to the kinetic energy of the arrow. Calculate the speed of the arrow as it leaves the bow. (c) The arrow is shot vertically upwards. Calculate the maximum height it reaches above the bow, assuming air resistance is negligible.

10. A wind turbine extracts kinetic energy from the wind. The air has a density of 1.2 kg m^{-3} and approaches the turbine with a speed of 9.5 m s^{-1}. The circular area swept out by the turbine blades is 1500 m^2. (a) Show that the mass of air passing through this area per second is approximately 1.7×10^4 kg s^{-1}. (b) Calculate the total kinetic energy of the air passing through this area per second. (c) The electrical power output of the turbine is 3.2×10^5 W. Calculate the efficiency of the wind turbine.

11. A force F acts on an object of mass 4.0 kg in a straight line. The variation of F with the displacement x of the object is shown in Fig. 11. (a) Use Fig. 11 to determine the total work done by the force when the object moves from $x = 0$ to $x = 12$ m. (b) The object was initially at rest at $x = 0$. Calculate the speed of the object at $x = 12$ m.

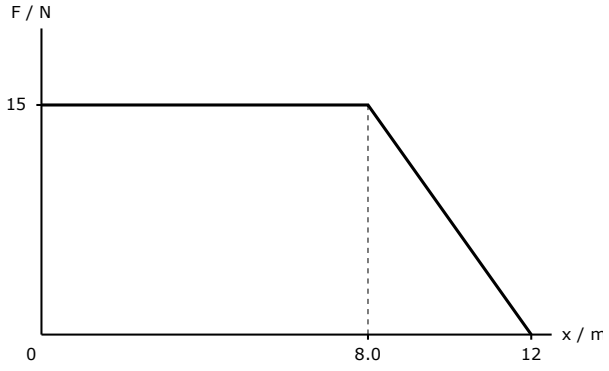

Fig. 11.

12. A roller coaster car of mass 850 kg passes point A at a speed of 12 m s^{-1}. Point A is at a vertical height of 18 m above the ground. The car descends to point B, which is at a vertical height of 5.0 m above the ground. The length of the track from A to B is 45 m. The average frictional force opposing the motion of the car between A and B is 1.2 kN. (a) Calculate the total energy (kinetic and gravitational potential) of the car at point A, relative to the ground. (b) Calculate the work done against friction as the car moves from point A to point B. (c) Use the principle of conservation of energy to determine the speed of the car at point B.

13. A block of mass 0.050 kg is projected vertically upwards by a compressed spring. The energy stored in the spring before release is 4.8 J. When released, 75% of this energy is transferred to the initial kinetic energy of the block. (a) Calculate the initial speed of the block. (b) Calculate the maximum height reached by the block above its point of release, assuming air resistance is negligible.

14. A train of mass 4.5×10^5 kg is travelling at a constant speed of 25 m s^{-1} on a horizontal track. The power output of the train's engine is 1.8 MW. (a) Calculate the total resistive force opposing the motion of the train. (b) The train now approaches an upward incline of 1.5° to the horizontal. The engine's power output remains constant and the resistive force is unchanged. Calculate the new constant speed of the train up the incline.

15. A bungee jumper of mass 65 kg falls from a tall bridge. After falling a vertical distance of 25 m, the bungee cord begins to stretch. Air resistance is negligible. (a) Calculate the kinetic energy of the jumper after falling 25 m. (b) The bungee cord stretches and brings the jumper to a momentary halt. The lowest point reached by the jumper is 65 m below the bridge. Determine the maximum elastic potential energy stored in the bungee cord.

16. An elevator (lift) of empty mass 600 kg carries 8 passengers, each of average mass 75 kg. The elevator is lifted through a height of 45 m in 35 s at a constant speed. (a) Calculate the total work done against gravity. (b) The electric motor lifting the elevator has an overall efficiency of 80%. Calculate the electrical power input to the motor.

17. A child of mass 32 kg slides down a playground slide. The child starts from rest at a height of 3.0 m above the ground and reaches the bottom of the slide with a speed of 4.5 m s^{-1}. (a) Calculate the work done against friction during the slide. (b) State what happens to the energy that is dissipated by friction.

18. (a) Define efficiency. (b) A petrol engine consumes fuel at a rate of 1.5×10^{-3} kg s^{-1}. The energy released by burning 1.0 kg of this fuel is 4.6×10^{7} J. The useful mechanical power output of the engine is 22 kW. (i) Calculate the total power input to the engine. (ii) Calculate the efficiency of the engine.

19. A car of mass 950 kg accelerates from rest to a speed of 28 m s^{-1} in a time of 8.5 s along a horizontal road. (a) Calculate the kinetic energy of the car at a speed of 28 m s^{-1}. (b) Calculate the average useful power output of the car's engine during this time. (c) The actual average power output of the engine is 65 kW. Suggest one reason why this is greater than the useful power output calculated in (b).

20. A motor drives a conveyor belt that transports 1500 kg of gravel per minute to the top of a hopper. The hopper is at a vertical height of 12 m above the loading point. The gravel starts from rest and is discharged at the top with a speed of 2.5 m s^{-1}. (a) Calculate the gain in gravitational potential energy of the gravel per minute. (b) Calculate the gain in kinetic energy of the gravel per minute. (c) The motor has an efficiency of 65%. Calculate the electrical power input to the motor.

21. A hydroelectric power station uses water falling through a vertical height of 45 m. The mass of water flowing through the turbines per second is 8.0×10^{4} kg s^{-1}. (a) Calculate the rate of decrease of gravitational potential energy of the water. (b) The electrical power output of the station is 28 MW. Calculate the efficiency of the power station.

22. A bullet of mass 8.0 g is fired horizontally at a speed of 450 m s^{-1} into a stationary wooden block. The bullet penetrates a distance of 12 cm into the block before coming to rest. (a) Calculate the initial kinetic energy of the bullet. (b) Use the principle of conservation of energy to determine the average resistive force exerted by the wood on the bullet.

23. A sports car is travelling on a horizontal road. At the instant when its speed is 20 m s^{-1}, the useful power delivered to the wheels is 120 kW and the total resistive force opposing the motion of the car is 1800 N. (a) Calculate the driving force provided by the engine at this instant. (b) The car is accelerating. Explain, in terms of forces, why the car is accelerating. (c) Calculate the rate at which the kinetic energy of the car is increasing at this instant.

24. A submarine of mass 4.2×10^6 kg is travelling horizontally at a steady speed of 8.0 m s^{-1} deep underwater. The engines produce a useful power of 3.5 MW. (a) Calculate the total drag force acting on the submarine. (b) The engines are turned off, and the submarine glides to a halt. Calculate the initial kinetic energy of the submarine. (c) The average drag force during the deceleration is half the value calculated in (a). Estimate the distance the submarine glides before coming to rest.

25. A catapult is used to launch a model glider of mass 0.45 kg horizontally. The catapult exerts a variable forward force on the glider over a distance of 1.2 m. The average value of this forward force is 65 N. A constant resistive force of 5.0 N acts on the glider during the launch. (a) Calculate the work done by the average forward force. (b) Calculate the work done against the resistive force during the launch. (c) Determine the speed of the glider at the instant it leaves the catapult.

Advanced Practice（培优）· 20

1. A constant resultant force F acts on an object of mass m, causing it to accelerate uniformly from an initial speed u to a final speed v over a displacement s. (a) Use the equation of motion relating v, u, the acceleration a and s to show that the work done by the force is equal to $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$. (b) Calculate the work done on a car of mass 1200 kg accelerating from 15 m s^{-1} to 25 m s^{-1}.

2. A cable car of mass 6500 kg is pulled up a steep mountain track by a motor at a constant speed of 4.0 m s^{-1}. The track has a total length of 1200 m and a vertical height of 550 m. The average resistive force due to friction opposing the motion of the cable car is 3500 N. (a) Calculate the gain in gravitational potential energy of the cable car when it reaches the top of the track. (b) Calculate the total work done against the resistive force. (c) Determine the useful power output of the motor pulling the cable car.

3. A small parcel of mass 1.2 kg slides down a spiral chute. It starts from rest at a vertical height of 4.5 m above the bottom of the chute. The total length of the path along the chute is 15 m. The parcel reaches the bottom with a speed of 6.0 m s^{-1}. (a) Calculate the loss of gravitational potential energy of the parcel. (b) Calculate the kinetic energy of the parcel at the bottom of the chute. (c) Determine the average frictional force acting on the parcel during its descent.

4. A conveyor belt transports 850 kg of crushed rock per minute from a quarry floor to a processing plant. The plant is at a vertical height of 24 m above the quarry floor. The rock is deposited into the plant with a speed of 1.5 m s^{-1}. The total electrical power input to the conveyor motor is 5.0 kW. (a) Calculate the rate of increase of gravitational potential energy of the rock. (b) Calculate the rate of increase of kinetic energy of the rock. (c) Determine the efficiency of the conveyor system.

5. A horizontal force F pushes a trolley of mass 15 kg in a straight line along a horizontal surface. The variation of F with displacement s is shown in Fig. 5. The trolley starts from rest at $s = 0$. Frictional forces are negligible. (a) Determine the total work done by the force F for the displacement from $s = 0$ to $s = 7.0$ m. (b) State the maximum kinetic energy of the trolley. (c) Calculate the final speed of the trolley at $s = 7.0$ m.

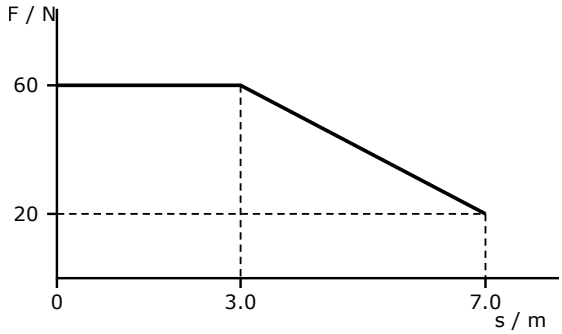

Fig. 5.

6. (a) By considering an object of mass m lifted vertically at constant velocity through a height Δh in a uniform gravitational field of strength g, derive the equation $\Delta E_P = mg\Delta h$. (b) A crane lifts a load of mass 850 kg vertically upwards. The load accelerates uniformly from rest to a speed of 1.5 m s^{-1} over a distance of 4.0 m. Calculate the total work done by the crane on the load.

7. A speedboat of mass 2200 kg is travelling at 24 m s^{-1} across a lake. The engine is switched off and a constant drag force brings the boat to rest in a distance of 85 m. (a) Calculate the initial kinetic energy of the boat. (b) Calculate the constant drag force acting on the boat. (c) If the boat was initially travelling at 12 m s^{-1} and the drag force remained the same, determine the new stopping distance without using equations of motion.

8. A wind turbine extracts kinetic energy from the wind. The air of density 1.2 kg m^{-3} flows towards the turbine at a speed of 12 m s^{-1}. The circular swept area of the turbine blades has a radius of 25 m. (a) Show that the mass of air passing through the swept area per second is 2.8×10^4 kg s^{-1}. (b) Calculate the total kinetic energy of the air passing through the swept area per second. (c) The turbine converts 40% of this kinetic energy into electrical energy. Calculate the electrical power output.

9. A skier of mass 72 kg starts from rest and skis down a straight slope of length 85 m. The slope drops a vertical height of 18 m. The skier experiences a constant resistive force of 45 N due to snow friction and air resistance. (a) Calculate the decrease in gravitational potential energy of the skier. (b) Calculate the work done against the resistive force. (c) Calculate the speed of the skier at the bottom of the slope.

10. (a) Power P is defined as the rate of work done. Derive the equation $P = Fv$ for an object moving at constant velocity v under a constant driving force F. (b) A cyclist and her bicycle have a combined mass of 75 kg. She is riding along a horizontal road at a constant speed of 8.0 m s^{-1}. The total resistive force F_R opposing her motion is given by the equation $F_R = kv^2$, where v is the speed and $k = 0.45$ kg m^{-1}. Calculate the resistive force acting on the cyclist. (c) Calculate the useful power output of the cyclist. (d) The cyclist now approaches a hill inclined at 3.0° to the horizontal. She maintains her useful power output. Explain why her speed must decrease, without further calculation.

11. A hammer of mass 1.5 kg is used to drive a nail into a wooden block. The hammer strikes the nail with a speed of 8.0 m s^{-1} and drives it 0.040 m downwards into the wood before coming to rest. The hammer remains in contact with the nail during this motion. (a) Calculate the kinetic energy of the hammer just before impact. (b) Calculate the loss of gravitational potential energy of the hammer as it moves 0.040 m downwards. (c) Use the principle of conservation of energy to determine the average resistive force exerted by the wood on the nail.

12. A drone flies horizontally at a constant speed of 12 m s^{-1}. It pulls a small advertising banner using a lightweight cable. The cable is at an angle of 15° below the horizontal, and the tension in the cable is 65 N. (a) Calculate the horizontal component of the tension in the cable. (b) Calculate the useful power output of the drone used to pull the banner. (c) The drone's battery supplies electrical energy at a rate of 2.4 kW, and the overall efficiency of the drone is 45%. Calculate the power used to keep the drone itself airborne and overcome its own air resistance.

13. A rocket of mass 0.25 kg is launched vertically upwards from rest. The thrust force F provided by the rocket engine is constant at 8.0 N for the first 15 m of its flight, after which the fuel is exhausted and the thrust becomes zero. Air resistance is negligible. (a) Calculate the work done by the thrust force. (b) Calculate the gain in gravitational potential energy of the rocket at the instant the fuel is exhausted. (c) Calculate the maximum height reached by the rocket above its launch position.

14. A bobsleigh of mass 630 kg is pushed along a horizontal ice track by its crew. The crew exerts a constant forward force of 950 N. The average resistive force acting on the bobsleigh is 180 N. The bobsleigh accelerates and reaches a speed of 8.5 m s^{-1} from rest. (a) Calculate the total work done on the bobsleigh by the resultant force. (b) Determine the distance over which the crew pushed the bobsleigh. (c) Calculate the total energy expended by the crew as work.

15. A geothermal power plant extracts 2500 kg of water per minute from an underground reservoir 120 m below the surface. The water is pumped up and ejected at the surface with a speed of 4.5 m s^{-1}. (a) Calculate the power required to increase the gravitational potential energy of the water. (b) Calculate the power required to increase the kinetic energy of the water. (c) The pump is driven by a motor with an electrical power input of 65 kW. Determine the efficiency of the pump system.

16. A stone of mass 0.20 kg is thrown vertically downwards from the top of a building of height 35 m with an initial speed of 8.0 m s^{-1}. Air resistance is negligible. (a) Calculate the initial kinetic energy of the stone. (b) Calculate the loss in gravitational potential energy of the stone as it falls to the ground. (c) Use the principle of conservation of energy to determine the speed of the stone just before it hits the ground.

17. A ski lift transports 65 skiers per hour from the base of a mountain to a station 450 m higher in vertical elevation. The average mass of a skier with their equipment is 85 kg. The lift travels at a constant speed along a cable of length 1800 m. (a) Calculate the useful work done in lifting one average skier to the station. (b) Calculate the useful power output of the ski lift motor required to transport the skiers. (c) Calculate the average driving force exerted by the lift on one skier along the direction of the cable.

18. A drone of mass 4.5 kg is hovering at a constant height. (a) State the useful work done by the drone's rotors while it is hovering. (b) The drone now climbs vertically upwards at a constant speed of 6.0 m s^{-1}. The air resistance opposing the motion is 5.5 N. Calculate the total upward force exerted by the rotors. (c) Calculate the useful power output of the drone's motors during the climb.

19. A bullet of mass 12 g is fired horizontally at a speed of 350 m s^{-1} into a stationary wooden block of thickness 0.080 m. The bullet emerges from the other side of the block with a speed of 120 m s^{-1}. (a) Calculate the loss of kinetic energy of the bullet. (b) Assuming the resistive force is constant, calculate the average resistive force exerted by the wooden block on the bullet. (c) The same bullet is fired at 350 m s^{-1} into a second identical wooden block but with a thickness of 0.15 m. Deduce whether the bullet will pass completely through the block.

20. A tidal barrage generates electricity by allowing sea water to flow through turbines as the tide falls. During a certain period, 4.5×10^6 kg of water flows through the turbines every second. The water effectively falls through a vertical height of 5.5 m. It enters the turbines at 2.0 m s^{-1} and leaves at 3.5 m s^{-1}. (a) Calculate the rate of loss of gravitational potential energy of the water. (b) Calculate the rate of gain of kinetic energy of the water. (c) Determine the net rate at which energy is transferred from the water to the turbines, and hence calculate the overall efficiency of the power station if the electrical power output is 180 MW.

6 Deformation of solids（固体的形变）

Study of the behavior of materials under tensile and compressive forces, including Hooke's law, stress, strain, the Young modulus, and elastic potential energy.

Rules 规则

- Hooke's law states that extension is directly proportional to the applied load, provided the limit of proportionality is not exceeded: $F = kx$ (胡克定律).
- Stress is the force applied per unit cross-sectional area: $\sigma = \frac{F}{A}$ (应力).
- Strain is the fractional increase in original length: $\varepsilon = \frac{x}{L}$ (应变).
- The Young modulus is the ratio of stress to strain in the linear elastic region: $E = \frac{\sigma}{\varepsilon} = \frac{FL}{Ax}$ (杨氏模量).
- Elastic deformation means the material returns to its original length when the load is removed (弹性形变); plastic deformation results in permanent extension (塑性形变).
- Elastic potential energy is the area under the force-extension graph; for the linear region, $E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$ (弹性势能).

Worked Examples 例题

Example 6.1

A spring has a spring constant of 400 N m^{-1}. Calculate the elastic potential energy stored when it is extended by 0.050 m within its limit of proportionality.

Solution

Use the formula for elastic potential energy: $E_P = \frac{1}{2}kx^2$. Substitute the given values: $E_P = \frac{1}{2} \times 400 \times (0.050)^2 = 0.50$ J.

Example 6.2

A steel wire of length 2.5 m and cross-sectional area 1.2×10^{-6} m^2 is subjected to a tensile force of 60 N. The wire extends by 0.60 mm. Calculate the Young modulus of the steel.

Solution

First, calculate the stress: $\sigma = \frac{F}{A} = \frac{60}{1.2 \times 10^{-6}} = 5.0 \times 10^7$ Pa. Next, calculate the strain: $\varepsilon = \frac{x}{L} = \frac{0.60 \times 10^{-3}}{2.5} = 2.4 \times 10^{-4}$. Finally, calculate the Young modulus: $E = \frac{\sigma}{\varepsilon} = \frac{5.0 \times 10^7}{2.4 \times 10^{-4}} = 2.1 \times 10^{11}$ Pa.

Example 6.3

A force-extension graph for a rubber band shows a non-linear curve. The area under the loading curve is 0.45 J and the area under the unloading curve is 0.30 J. State what the area under the loading curve represents and calculate the thermal energy dissipated during one cycle.

Solution

The area under the loading curve represents the work done on the rubber band (or the total elastic potential energy stored during stretching). The thermal energy dissipated is the difference between the work done stretching the band and the work done by the band when unloading: $0.45 - 0.30 = 0.15$ J.

Example 6.4

A wire of length 3.0 m and diameter 0.80 mm is tested. The gradient of the linear section of its force-extension graph is 2.5×10^4 N m^{-1}. Determine the Young modulus of the material.

Solution

First, calculate the cross-sectional area $A = \frac{\pi d^2}{4} = \frac{\pi \times (0.80 \times 10^{-3})^2}{4} = 5.03 \times 10^{-7}$ m^2. The Young modulus is given by $E = \frac{FL}{Ax}$, which can be written as $E = \text{gradient} \times \frac{L}{A}$. Substituting the values: $E = (2.5 \times 10^4) \times \frac{3.0}{5.03 \times 10^{-7}} = 1.5 \times 10^{11}$ Pa.

考点总览 Key Points

- ☐ 我理解拉力 (tensile force) 与压力 (compressive force) 会引起物体的一维形变 (deformation)。
- ☐ 我掌握载荷 (load)、伸长量 (extension)、压缩量 (compression) 及正比极限 (limit of proportionality) 的概念。
- ☐ 我能熟练运用胡克定律 (Hooke's law) 及劲度系数公式进行计算（$k = \frac{F}{x}$）。
- ☐ 我会定义并计算应力 (stress)、应变 (strain) 以及材料的杨氏模量 (Young modulus, $E = \frac{\sigma}{\varepsilon}$)。
- ☐ 我熟悉测定金属丝杨氏模量 (Young modulus) 的实验设计、测量步骤与数据处理方法。
- ☐ 我能区分弹性形变 (elastic deformation) 与塑性形变 (plastic deformation)，并理解弹性极限 (elastic limit)。
- ☐ 我理解力-伸长量图像 (force–extension graph) 下方的面积代表外力所做的功 (work done)。
- ☐ 我掌握在正比极限内计算弹性势能 (elastic potential energy) 的公式（$E_p = \frac{1}{2}Fx = \frac{1}{2}kx^2$）及图像求法。

Basic Drill（入门综合）· 50

1. Define tensile stress.

2. A steel wire of original length 2.000 m is stretched by a tensile force so that its extension is 3.0 mm. Calculate the tensile strain in the wire.

3. A helical spring extends by 0.050 m when a tensile load of 15 N is applied within its limit of proportionality. Calculate the spring constant of the spring.

4. State Hooke's law.

5. A metal wire of cross-sectional area 1.5×10^{-6} m^2 is subjected to a tensile force of 120 N. Calculate the tensile stress in the wire.

6. A spring has a spring constant of 250 N m^{-1}. Calculate the extension produced by a tensile load of 40 N, assuming Hooke's law is obeyed.

7. A wire subjected to a tensile stress of 1.8×10^8 Pa experiences a tensile strain of 1.5×10^{-3}. Calculate the Young modulus of the material of the wire.

8. A spring with a spring constant of 400 N m^{-1} is stretched by an extension of 0.020 m within its limit of proportionality. Calculate the elastic potential energy stored in the spring.

9. State the measuring instrument used to determine the diameter of a thin wire accurately in an experiment to measure its Young modulus.

10. A compression spring has a spring constant of 800 N m^{-1}. Calculate the compressive force needed to compress the spring by 15 mm.

11. Define the elastic limit of a material.

12. A nylon cord has an unstretched length of 1.50 m. When a load is hung from it, its new length is 1.53 m. Calculate the strain in the cord.

13. A tensile force of 60 N extends a metal wire by 4.0 mm within its limit of proportionality. Calculate the strain energy stored in the wire.

14. Distinguish between elastic deformation and plastic deformation.

15. A cylindrical metal wire has a diameter of 0.40 mm. Calculate the cross-sectional area of the wire.

16. A metal wire of original length 2.5 m and uniform cross-sectional area 2.0×10^{-7} m^2 extends by 3.0 mm when a tensile force of 50 N is applied. Calculate the Young modulus of the metal.

17. State what is represented by the area under a force–extension graph for a material undergoing deformation.

18. A vertical spring extends by 12 mm when a load of 6.0 N is suspended from its lower end. Calculate the energy stored in the spring, assuming Hooke's law is obeyed.

19. A spring of spring constant 320 N m^{-1} stores an elastic potential energy of 0.40 J within its limit of proportionality. Calculate the extension of the spring.

20. A tensile force of 450 N produces a tensile stress of 3.0×10^7 Pa in a metal bar. Calculate the cross-sectional area of the bar.

21. State what is meant by the limit of proportionality.

22. A spring storing an elastic potential energy E when stretched by an extension x is stretched to an extension of $2x$. The limit of proportionality is not exceeded. In terms of E, state the new elastic potential energy stored in the spring.

23. A copper wire of length 2.0 m and cross-sectional area 5.0×10^{-7} m^2 has a Young modulus of 1.1×10^{11} Pa. Calculate the extension of the wire when a tensile load of 22 N is applied.

24. A compressive force of 5.0 kN is applied uniformly to the top of a vertical pillar with a cross-sectional area of 0.025 m^2. Calculate the compressive stress in the pillar.

25. In an experiment to determine the Young modulus of a metal wire, explain why a parallel reference wire is used alongside the test wire.

26. State Hooke's law.

27. A helical spring has a spring constant of 80 N m^{-1}. Calculate the extension of the spring when a tensile force of 12 N is applied within the limit of proportionality.

28. A steel rod has a cross-sectional area of 2.5×10^{-5} m^2 and a Young modulus of 2.0×10^{11} Pa. Calculate the tensile force required to produce a strain of 6.0×10^{-4} in the rod.

29. A helical spring has an unstretched length of 12.0 cm. When a tensile force of 18 N is applied within the limit of proportionality, its length increases to 16.5 cm. Calculate the spring constant of the spring.

30. A copper wire has a uniform cross-sectional area of 1.2×10^{-6} m^2. A tensile force of 60 N is applied to the wire. Calculate the tensile stress in the wire.

31. A steel cable of original length 4.0 m extends by 2.4 mm when supporting a load. Calculate the tensile strain in the cable.

32. State the SI base units of the Young modulus.

33. Define the Young modulus of a material.

34. A metal wire of original length 1.80 m and uniform cross-sectional area 4.50×10^{-7} m^2 extends by 0.800 mm under a load of 90.0 N within its limit of proportionality. Calculate the Young modulus of the metal.

35. State what is meant by plastic deformation.

36. State what physical quantity is represented by the area under a force–extension graph.

37. A spring obeying Hooke's law extends by 0.060 m when a tensile force of 15 N is applied. Calculate the elastic potential energy stored in the spring.

38. A wire has a spring constant of 4.0×10^4 N m^{-1}. Calculate the elastic potential energy stored in the wire when it is extended by 3.0 mm within the limit of proportionality.

39. In an experiment to determine the Young modulus of a metal wire, suggest why the diameter of the wire is measured at several different positions along its length.

40. When a rubber band is stretched and then allowed to contract, the loading and unloading curves form a closed loop. State what is represented by the area enclosed by this loop.

41. A brass wire has a circular cross-section with a diameter of 0.80 mm. Calculate the cross-sectional area of the wire in m^2.

42. A solid cylinder of original length 50.0 mm is compressed along its axis so that its length decreases to 48.5 mm. Calculate the compressive strain of the cylinder.

43. State the distinction between the limit of proportionality and the elastic limit.

44. A metal wire with a Young modulus of 2.0×10^{11} Pa is subjected to a tensile stress of 1.6×10^8 Pa. Calculate the resulting tensile strain in the wire.

45. Two identical springs, each having a spring constant of 120 N m^{-1}, are connected in parallel. Calculate the effective spring constant of this parallel combination.

46. A wire obeying Hooke's law stores 0.064 J of elastic potential energy when a tensile force of 80 N is applied. Calculate the extension of the wire.

47. A force F extends a spring by an extension x, storing elastic potential energy E_p. The extension is increased to $3x$ within the limit of proportionality. Determine the new elastic potential energy stored in the spring in terms of E_p.

48. In an experiment to determine the Young modulus of a metal wire, suggest why a long wire is used rather than a short wire.

49. A structural support rod of uniform cross-sectional area 2.0×10^{-4} m^2 is subjected to a compressive force of 1.4×10^4 N. Calculate the compressive stress in the rod.

50. The graph shows the variation of tensile force F with extension x for a metal wire up to its limit of proportionality at $F = 80$ N and $x = 2.0$ mm. Calculate the work done in stretching the wire to this extension.

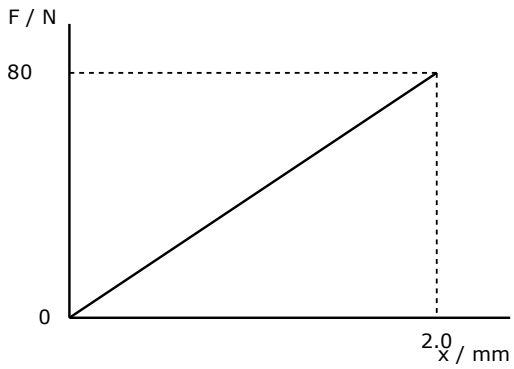

Fig. 50. Graph of tensile force against extension for the metal wire.

Practice（综合练习）· 25

1. A spring of unstretched length 0.450 m is suspended vertically. A mass of 2.50 kg is attached to the lower end, and the spring extends to a total length of 0.580 m. Calculate the spring constant of the spring.

2. A solid cylindrical pillar of height 2.40 m and cross-sectional area 0.0550 m^2 supports a compressive load of 4.20×10^5 N. The material of the pillar has a Young modulus of 3.80×10^{10} Pa. Calculate the compression of the pillar.

3. (a) State Hooke's law. (b) Define the limit of proportionality.

4. A force-extension graph for a metal wire is linear up to a force of 45 N and an extension of 3.0 mm. Calculate the elastic potential energy stored in the wire when it is extended by 3.0 mm.

5. A steel wire of diameter 0.80 mm and length 3.5 m is subjected to a tensile force of 65 N. The Young modulus of steel is 2.1×10^{11} Pa. Calculate: (a) the stress in the wire (b) the strain in the wire.

6. In an experiment to determine the Young modulus of a metal wire, a student measures the original length of the wire as 2.500 m and its diameter as 0.56 mm. A load of 30.0 N produces an extension of 1.8 mm. (a) Calculate the Young modulus of the metal. (b) Suggest one suitable instrument that could be used to measure the diameter of the wire.

7. A tensile force F is applied to a metal wire. The corresponding extension x is measured. The data are recorded below: F / N: 0, 15.0, 30.0, 45.0, 60.0, 75.0 x / mm: 0, 1.20, 2.40, 3.60, 5.20, 7.20 (a) State the maximum force shown in the table at which the wire still obeys Hooke's law. (b) Calculate the spring constant of the wire in the region where Hooke's law is obeyed. (c) The force-extension graph for the wire can be modeled as straight-line segments connecting the data points. Calculate the total work done in stretching the wire from $x = 0$ to $x = 7.20$ mm.

8. A spring obeys Hooke's law. When it is stretched by 2.0 cm, the elastic potential energy stored in the spring is 0.15 J. Calculate the total elastic potential energy stored when the spring is stretched by 6.0 cm.

9. A spring has a spring constant of 450 N m^{-1}. Calculate the work done in stretching the spring from an extension of 0.020 m to an extension of 0.050 m.

10. Wire A and wire B are made of the same material. Wire A has twice the unstretched length and half the diameter of wire B. Both wires are subjected to the same tensile force and obey Hooke's law. Determine the ratio $\frac{\text{extension of wire A}}{\text{extension of wire B}}$.

11. A nylon rope has a breaking stress of 7.5×10^7 Pa. The rope has a uniform circular cross-section of diameter 1.2 cm. Calculate the maximum load that the rope can support before breaking.

12. The graph in Fig. 12 shows the variation with extension x of the force F applied to a wire. (a) Use the graph to determine the spring constant of the wire in its linear region. (b) State the extension at the limit of proportionality.

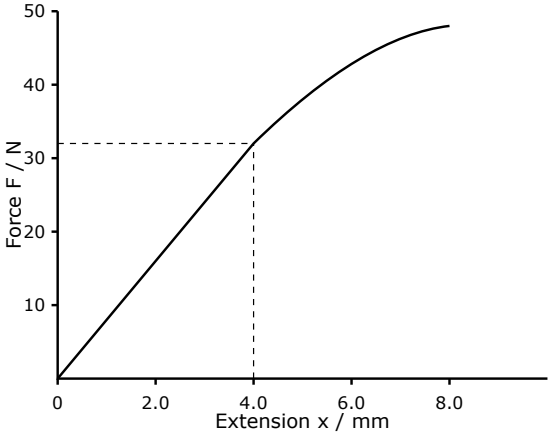

Fig. 12.

13. A copper wire of length 1.80 m and cross-sectional area 3.20×10^{-7} m^2 is suspended vertically. A mass of 4.00 kg is attached to its lower end. The Young modulus of copper is 1.30×10^{11} Pa. Calculate the elastic potential energy stored in the wire.

14. A rubber cord is stretched by an applied force. The force-extension graph forms a closed loop. The area under the loading curve is 0.85 J and the area under the unloading curve is 0.60 J. (a) State what the area under the loading curve represents. (b) Calculate the thermal energy dissipated during one complete load-unload cycle.

15. The stress-strain graph for a metallic wire is a straight line through the origin up to a stress of 1.5×10^8 Pa and a strain of 0.0012. Calculate the Young modulus of the metal.

16. A composite wire consists of a 2.00 m length of aluminium joined end-to-end to a 3.00 m length of steel. Both wires have a diameter of 1.00 mm. A tensile force of 150 N is applied to the composite wire. The Young modulus of aluminium is 7.0×10^{10} Pa and of steel is 2.1×10^{11} Pa. Calculate the total extension of the composite wire.

17. A metal rod of volume 4.5×10^{-5} m^3 is subjected to a tensile stress of 8.0×10^7 Pa. The strain produced is 4.0×10^{-4}. Assuming Hooke's law is obeyed, calculate the total elastic potential energy stored in the rod.

18. A mass of 0.80 kg is placed on top of a vertical spring, causing it to compress by 4.5 cm. The spring obeys Hooke's law. Calculate: (a) the spring constant (b) the elastic potential energy stored in the compressed spring.

19. A titanium wire of cross-sectional area 2.4×10^{-6} m^2 is stretched by a force of 180 N. The extension produced is 0.45 mm. The Young modulus of titanium is 1.1×10^{11} Pa. Calculate the original length of the wire.

20. A uniform elastic cord has an unstretched length of 1.00 m and a spring constant of 50.0 N m^{-1}. The cord is cut into five equal pieces, each of length 0.200 m. These five pieces are then arranged side-by-side in parallel to form a single composite cord of unstretched length 0.200 m. (a) Deduce the spring constant of the composite cord. (b) A tensile force of 150 N is applied to the composite cord within its limit of proportionality. Calculate the extension produced. (c) Calculate the elastic potential energy stored in the composite cord under this force.

21. A wire has a force-extension graph that is linear up to a force of 40 N and an extension of 4.0 mm. The wire is further stretched to a maximum force of 55 N at an extension of 7.0 mm. When the force is removed, the wire recovers along a line parallel to its initial linear region. Calculate the permanent extension of the wire.

22. A solid brick of mass 2.50 kg rests on a rubber cylinder of unstretched height 5.00 cm and diameter 3.00 cm. The Young modulus of the rubber is 1.50×10^6 Pa. Calculate the compressive strain in the rubber cylinder.

23. The force-extension graph for a material consists of a straight line from the origin $(0, 0)$ to a point where the force is 20 N and the extension is 2.0 mm. The graph then continues as a horizontal line to an extension of 5.0 mm. Calculate the total work done in stretching the material to an extension of 5.0 mm.

24. A student carries out an experiment to determine the Young modulus of a metal wire. (a) State four physical quantities that the student must measure. (b) Explain how the Young modulus is determined from a graph of applied force against extension.

25. A spring is compressed by a force of 50 N. The elastic potential energy stored in the spring is 1.2 J. Assuming the spring obeys Hooke's law, calculate: (a) the compression of the spring (b) the spring constant.

Advanced Practice（培优）· 20

1. A steel wire of length 2.0 m and diameter 1.2 mm is joined end-to-end to a brass wire of length 1.5 m and diameter 1.6 mm. The composite wire is suspended vertically and a load of 450 N is applied. The Young modulus of steel is 210 GPa and of brass is 100 GPa. (a) Calculate the total extension of the composite wire. (b) Calculate the total elastic potential energy stored in the wires.

2. (a) Show that the elastic potential energy E_P stored in a wire of cross-sectional area A, original length L, and Young modulus E, when extended by x within its limit of proportionality, is given by $E_P = \frac{EAx^2}{2L}$. (b) A copper wire of length 3.5 m and cross-sectional area 4.0×10^{-6} m^2 is stretched by 2.4 mm. The Young modulus of copper is 130 GPa. Calculate the elastic potential energy stored per unit volume in the wire.

3. The force-extension graph for a metal wire is shown in the figure. The graph is linear up to a force of 60 N, and then follows a different straight line up to 80 N. The wire is loaded to 80 N and then unloaded. The unloading line is parallel to the initial linear section of the loading graph. (a) Determine the permanent extension of the wire after it has been fully unloaded. (b) Calculate the thermal energy dissipated during this loading and unloading cycle.

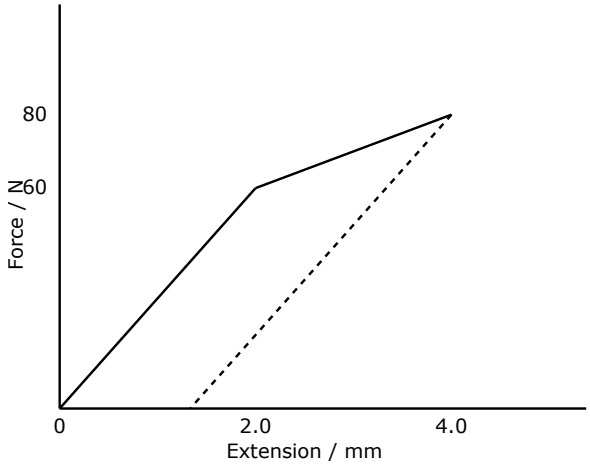

Fig. 3. Force-extension graph for the metal wire

4. A rigid horizontal plate is supported by two vertical wires, A and B, each of unstretched length 2.5 m. Wire A is made of steel (Young modulus 210 GPa, cross-sectional area 2.0 mm^2) and wire B is made of aluminium (Young modulus 70 GPa, cross-sectional area 6.0 mm^2). A downward vertical force F is applied to the plate such that both wires extend equally by 1.5 mm. (a) Calculate the force F. (b) Determine the ratio of the stress in wire A to the stress in wire B.

5. A uniform spring has an unstretched length of 0.60 m and a spring constant of 150 N m^{-1}. The spring is cut into two pieces, A and B. The unstretched length of piece A is 0.20 m and the unstretched length of piece B is 0.40 m. (a) Determine the spring constant of piece A and the spring constant of piece B. (b) The two pieces are now connected in parallel and a load of 45 N is applied to the combination. Calculate the extension of the parallel combination.

6. A rubber cord is stretched and then relaxed, forming a hysteresis loop as shown in the force-extension graph. (a) Calculate the work done to stretch the rubber cord to an extension of 50 mm. (b) Calculate the energy dissipated as heat in one complete loading and unloading cycle.

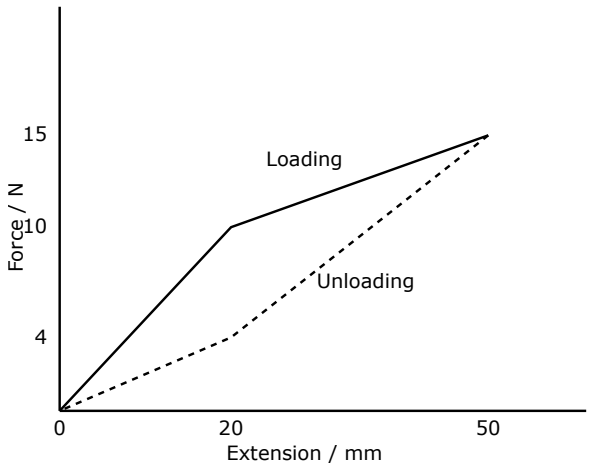

Fig. 6. Force-extension hysteresis loop for a rubber cord

7. In an experiment to determine the Young modulus of a metal, a student measures the extension of a wire for different loads. The wire has a diameter of 0.46 mm and an original length of 2.45 m. The gradient of the resulting load-extension graph is 1.4×10^4 N m^{-1}. (a) Calculate the Young modulus of the metal. (b) Suggest how the gradient of the graph would change if a wire of the same material but twice the diameter and half the length were used.

8. A glass fibre has a diameter of 0.15 mm and a length of 1.2 m. The breaking stress of the glass is 3.2×10^9 Pa and its Young modulus is 70 GPa. The glass behaves purely elastically up to its breaking point. (a) Calculate the force required to break the fibre. (b) Calculate the maximum elastic potential energy stored in the fibre before it breaks.

9. Spring 1 has a spring constant of 200 N m^{-1} and Spring 2 has a spring constant of 300 N m^{-1}. The two springs are connected in series and a tensile force of 60 N is applied to the combination. (a) Calculate the total extension of the combination. (b) Show that the ratio of the elastic potential energy stored in Spring 1 to that stored in Spring 2 is 1.5.

10. A concrete column of height 4.0 m and cross-sectional area 0.15 m^2 supports a compressive load of 8.5×10^5 N. The Young modulus of concrete is 30 GPa. (a) Calculate the compressive stress and the compressive strain in the column. (b) Calculate the decrease in height of the column.

11. The force-extension graph for a sample of material is shown. The material is stretched to an extension of 12.0 mm and then unloaded. The unloading curve is a straight line. (a) Estimate the work done in stretching the material to 12.0 mm. (b) Calculate the energy recovered when the material is completely unloaded.

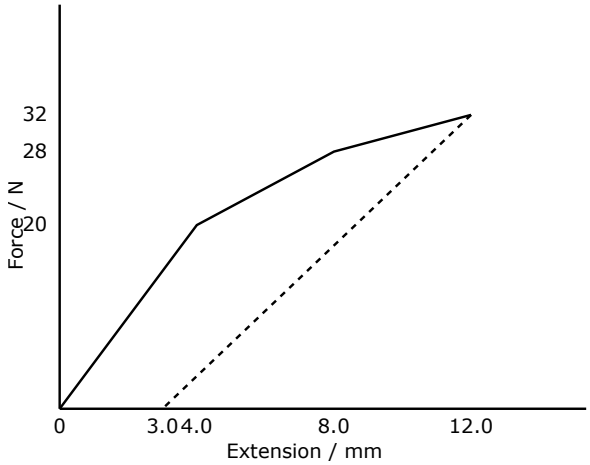

Fig. 11. Force-extension graph for the material

12. A metal wire A has length L and diameter d. A second wire B of the same material has length $3L$ and diameter $2d$. Both wires are subjected to the same tensile force F and obey Hooke's law. (a) Determine the ratio of the extension of wire A to the extension of wire B. (b) Determine the ratio of the strain in wire A to the strain in wire B.

13. A nylon rope of length 15 m and diameter 12 mm is used to tow a car. The tension in the rope is 2500 N. The Young modulus of nylon is 2.7 GPa. (a) Calculate the elastic potential energy stored in the rope. (b) The rope suddenly snaps. Explain why this stored energy makes the snapping rope dangerous.

14. A wire has a limit of proportionality at a load of 40 N and an extension of 2.5 mm. It has an elastic limit at 45 N and an extension of 3.0 mm. (a) Calculate the spring constant of the wire within its limit of proportionality. (b) Assuming the force-extension graph is a straight line between the limit of proportionality and the elastic limit, calculate the total work done in stretching the wire from zero extension to its elastic limit.

15. A composite pillar consists of a hollow steel cylinder of external diameter 100 mm and internal diameter 80 mm, completely filled with a concrete core. The pillar has a length of 2.5 m and is compressed by a vertical force of 5.0×10^5 N. The Young modulus of steel is 200 GPa and of concrete is 25 GPa. (a) Calculate the cross-sectional area of the steel cylinder and the concrete core. (b) By equating the compressive strain in both materials, determine the force supported by the steel cylinder.

16. A single wire is stretched by a load of 120 N, producing an extension of 1.5 mm. A second identical wire is placed in parallel with the first wire. The same total load of 120 N is applied to the combination. (a) Calculate the new extension of the parallel combination. (b) Calculate the total elastic potential energy stored in the parallel combination and compare it to the energy stored in the single wire.

17. A force-extension graph for a material is a straight line passing through the origin. (a) Explain how the graph shows that the material obeys Hooke's law. (b) Use the graph to show that the elastic potential energy E_P is given by $E_P = \frac{1}{2}kx^2$, where k is the spring constant and x is the extension. (c) A spring has $k = 850$ N m^{-1}. Calculate the work done to stretch it from an extension of 2.0 cm to 5.0 cm.

18. A wire of total length 3.0 m consists of two sections. The first 2.0 m section has a diameter of 1.5 mm, and the remaining 1.0 m section has a diameter of 1.0 mm. The material has a Young modulus of 110 GPa. A tension of 150 N is applied to the wire. (a) Calculate the extension of the first section of the wire. (b) Calculate the total extension of the wire.

19. (a) Two identical springs, each of spring constant k, are connected in series. Show that the effective spring constant of the combination is $k/2$. (b) A spring of unstretched length 0.40 m and spring constant 300 N m^{-1} is cut into two equal halves. The two halves are then connected in parallel. Calculate the effective spring constant of this parallel combination.

20. The stress-strain graph for a material is shown. (a) Calculate the Young modulus of the material. (b) The material is formed into a wire of length 4.0 m and cross-sectional area 3.0×10^{-6} m^2. Calculate the extension of this wire when a load of 600 N is applied.

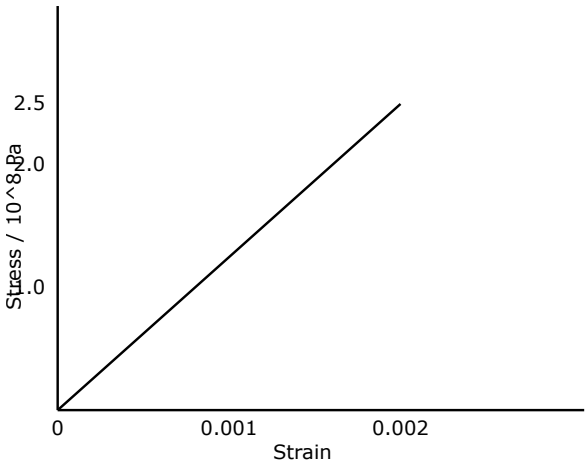

Fig. 20. Stress-strain graph for the material

7 Waves（波）

Properties of progressive waves, Doppler effect for sound, the electromagnetic spectrum, and polarisation.

Rules 规则

- The wave equation relates speed, frequency, and wavelength: $v = f\lambda$ (波速公式).
- Intensity is power per unit area, $I = \frac{P}{A}$, and is directly proportional to the square of the amplitude, $I \propto A^2$ (强度与振幅的平方成正比).
- Doppler effect for a moving source: $f_0 = \frac{f_s v}{v \pm v_s}$. Use minus for an approaching source and plus for a receding source (多普勒效应).
- All electromagnetic waves travel at $c = 3.0 \times 10^8$ m s^{-1} in a vacuum; visible light wavelengths range from 400 nm to 700 nm (电磁波谱与可见光).
- Malus's law for plane-polarised waves: $I = I_0 \cos^2\theta$, where θ is the angle between the polarisation plane and the transmission axis (马吕斯定律).
- On a cathode-ray oscilloscope (CRO), period T = time-base × horizontal divisions, and amplitude A = y-gain × vertical divisions (示波器).

Worked Examples 例题

Example 7.1

A progressive wave has an amplitude A and an intensity I. The amplitude of the wave is increased to $3A$ while its frequency remains constant. State the new intensity in terms of I. Also calculate the wavelength of the wave if its speed is 340 m s^{-1} and its frequency is 500 Hz.

Solution

Intensity is proportional to the square of the amplitude, $I \propto A^2$. If the amplitude is tripled ($3A$), the new intensity is $(3)^2 = 9$ times the original, giving $9I$. The wavelength is calculated using the wave equation $v = f\lambda$. Rearranging gives $\lambda = \frac{v}{f} = \frac{340}{500} = 0.68$ m.

Example 7.2

A car horn emits a sound of frequency 450 Hz. The car moves directly towards a stationary observer at a constant speed of 25 m s^{-1}. The speed of sound in air is 340 m s^{-1}. Calculate the frequency of the sound heard by the observer.

Solution

Use the Doppler effect equation for a source approaching a stationary observer: $f_0 = \frac{f_s v}{v - v_s}$. Substitute the given values: $f_0 = \frac{450 \times 340}{340 - 25} = \frac{153000}{315} = 485.7$ Hz. To three significant figures, the observed frequency is 486 Hz.

Example 7.3

A plane-polarised beam of light of intensity I_0 is incident normally on a polarising filter. The transmission axis of the filter is at an angle of 30° to the plane of polarisation of the incident light. Calculate the transmitted intensity in terms of I_0.

Solution

Use Malus's law to find the transmitted intensity: $I = I_0 \cos^2 \theta$. Substitute the angle $\theta = 30°$: $I = I_0 \cos^2(30°) = I_0 \left(\frac{\sqrt{3}}{2}\right)^2 = 0.75 I_0$.

Example 7.4

A sound wave is detected by a microphone connected to a cathode-ray oscilloscope (CRO). The time-base setting of the CRO is 2.0 ms cm^{-1}. The horizontal distance between two consecutive peaks on the screen is 4.0 cm. Calculate the frequency of the sound wave.

Solution

The period T is the time taken for one complete oscillation. T = time-base × horizontal distance = 2.0 ms cm^{-1} × 4.0 cm = 8.0 ms = 8.0×10^{-3} s. The frequency f is the reciprocal of the period: $f = \frac{1}{T} = \frac{1}{8.0 \times 10^{-3}} = 125$ Hz.

考点总览 Key Points

- ☐ 我理解行波 (progressive wave) 传递能量的过程，并能准确使用位移 (displacement)、振幅 (amplitude)、波长 (wavelength) 及相位差 (phase difference) 等概念。
- ☐ 我会利用阴极射线示波器 (CRO) 的时基 (time-base) 和 Y 增益 (y-gain) 确定波的振幅与频率 (frequency)。
- ☐ 我能推导波动方程，并熟练运用公式计算波速、频率和波长（$v = f\lambda$）。
- ☐ 我掌握波强度的概念，并能熟练应用强度公式及强度与振幅的关系进行计算（$I = \frac{P}{A}$ 且 $I \propto A^2$）。
- ☐ 我能比较横波 (transverse wave) 与纵波 (longitudinal wave) 的振动与传播特征，并熟练分析两者的图像表示。
- ☐ 我理解声源移动时的多普勒效应 (Doppler effect)，并能熟练计算观察者接收到的频率（$f_o = \frac{f_s v}{v \pm v_s}$）。
- ☐ 我熟悉电磁波谱 (electromagnetic spectrum) 各波段的波长范围，知晓其均为横波且真空中光速相同（可见光为 400–700 nm）。
- ☐ 我掌握偏振 (polarisation) 是横波特有现象的原理，并能运用马吕斯定律 (Malus's law) 计算偏振光的强度（$I = I_0 \cos^2\theta$）。

Basic Drill（入门综合）· 50

1. A sound wave in air has a frequency of 440 Hz and travels at a speed of 330 m s^{-1}. Calculate the wavelength of the sound wave.

2. A progressive transverse wave has a wavelength of 0.40 m and a period of 0.020 s. Calculate the speed of the wave.

3. The trace of a sinusoidal sound wave is displayed on a cathode-ray oscilloscope (CRO). The time-base of the CRO is set to 0.50 ms cm^{-1}. One complete cycle of the wave occupies a horizontal distance of 4.0 cm on the screen. Calculate the frequency of the sound wave.

4. An alternating electrical signal is displayed on a CRO screen. The Y-gain is set to 2.5 V cm^{-1}. The vertical distance from a peak to an adjacent trough of the trace is 4.8 cm. Determine the amplitude of the signal.

5. A progressive wave transfers energy with an intensity I. The amplitude of the wave is increased from 2.0 mm to 6.0 mm while its frequency remains constant. Determine the ratio $\frac{\text{new intensity}}{\text{original intensity}}$.

6. A loudspeaker emits sound uniformly in all directions with a total power of 18 W. Calculate the intensity of the sound wave at a distance of 3.0 m from the loudspeaker.

7. Two points along the path of a progressive wave are separated by a distance equal to $\frac{3}{8}$ of the wavelength. Calculate the phase difference between the oscillations of these two points, in degrees.

8. State the fundamental difference between a transverse wave and a longitudinal wave in terms of the direction of particle oscillations relative to the direction of energy propagation.

9. State whether each of the following waves is transverse or longitudinal: (a) ultrasound waves in water, (b) microwave radiation in a vacuum.

10. A displacement–distance graph for a transverse wave traveling at 15 m s^{-1} shows that the distance between adjacent crests is 0.60 m. Calculate the frequency of the wave.

11. A displacement–time graph for a particle in the path of a progressive wave shows that the time interval between a crest and the consecutive trough is 0.040 s. Calculate the period and the frequency of the wave.

12. An ambulance sounding a siren of frequency 720 Hz travels at a constant speed of 28 m s^{-1} directly towards a stationary observer. The speed of sound in air is 340 m s^{-1}. Calculate the frequency of the sound heard by the observer.

13. A train moving away from a stationary observer at 20 m s^{-1} sounds a horn of frequency 450 Hz. The speed of sound in air is 340 m s^{-1}. Calculate the frequency heard by the observer.

14. A car horn emits sound of frequency 500 Hz. A stationary observer detects a frequency of 530 Hz as the car approaches. The speed of sound in air is 340 m s^{-1}. Calculate the speed of the car.

15. A radio station broadcasts electromagnetic waves of frequency 100 MHz in free space. The speed of light in free space is 3.00×10^8 m s^{-1}. Calculate the wavelength of the radio waves.

16. State the principal region of the electromagnetic spectrum to which electromagnetic radiation with a wavelength of 5.5×10^{-7} m belongs.

17. State the approximate range of wavelengths in free space, in nanometres (nm), of electromagnetic waves visible to the human eye.

18. An electromagnetic wave has a wavelength of 0.030 m in free space. State the region of the electromagnetic spectrum to which it belongs and calculate its frequency.

19. Explain why sound waves cannot be plane-polarised.

20. A beam of plane-polarised light of intensity 60 W m^{-2} is incident normally on a polarising filter. The transmission axis of the filter is at an angle of 30° to the plane of polarisation of the light. Calculate the intensity of the transmitted light.

21. A plane-polarised light wave of intensity I_0 passes through a polarising filter. The intensity of the light transmitted through the filter is $0.25I_0$. Calculate the angle between the transmission axis of the filter and the plane of polarisation of the incident light.

22. A laser emits a uniform beam of light with a power of 15 mW. The cross-sectional area of the beam is 3.0×10^{-6} m^2. Calculate the intensity of the beam.

23. A source of sound waves moves towards a stationary observer. Explain why the frequency heard by the observer is higher than the frequency emitted by the source.

24. A beam of plane-polarised light is incident normally on a polarising filter whose transmission axis is oriented at an angle of 90° to the direction of polarisation of the light. State the intensity of the transmitted light.

25. A progressive wave travels along a rope. Two points on the rope are separated by a distance of 0.15 m. The wavelength of the wave is 0.60 m. Determine the phase difference between the two points in radians.

26. An ultrasonic progressive wave travels through water at a speed of 1500 m s^{-1}. The frequency of the wave is 2.5 MHz. Calculate the wavelength of the wave in water.

27. A plane-polarised beam of light of intensity 48 W m^{-2} is incident normally on a polarising filter. The transmission axis of the filter is at an angle of 60° to the plane of polarisation of the incident light. Calculate the intensity of the transmitted light.

28. A sinusoidal wave is displayed on a cathode-ray oscilloscope (CRO). The vertical distance between a peak and an adjacent trough on the screen is 4.0 cm. The Y-gain setting of the CRO is 0.50 V cm^{-1}. Determine the amplitude of the signal.

29. State the direction of the oscillations of particles in a medium relative to the direction of energy propagation for a longitudinal wave.

30. An ambulance travels directly away from a stationary observer at a constant speed of 28.0 m s^{-1}. The siren emits sound of frequency 680 Hz. The speed of sound in air is 340 m s^{-1}. Calculate the frequency of the sound heard by the observer.

31. Monochromatic visible light has a wavelength of 540 nm in free space. The speed of light in free space is 3.00×10^8 m s^{-1}. Calculate the frequency of this light.

32. A progressive wave has an initial intensity I and amplitude A. The amplitude of the wave is increased to $2.5A$. Determine the new intensity of the wave in terms of I.

33. A progressive transverse wave of wavelength 0.80 m travels along a string. Two points on the string are separated by a distance of 0.20 m along the direction of wave travel. Calculate the phase difference between the oscillations of these two points.

34. A plane-polarised beam of light is incident normally on a polarising filter. The transmitted intensity is equal to 50% of the incident intensity. Calculate the acute angle between the transmission axis of the filter and the plane of polarisation of the incident light.

35. A police car sounding a horn of frequency 750 Hz approaches a stationary pedestrian at a constant speed v_s. The pedestrian detects a frequency of 800 Hz. The speed of sound in air is 340 m s^{-1}. Calculate the speed v_s of the car.

36. State what is transferred by a progressive wave without transferring matter.

37. A small source emits sound waves uniformly in all directions at a constant power of 18.0 W. Calculate the sound intensity at a distance of 3.00 m from the source.

38. A signal is displayed on a cathode-ray oscilloscope (CRO). One complete cycle of the waveform occupies a horizontal distance of 5.0 cm on the screen. The time-base setting is 2.0 ms cm^{-1}. Calculate the frequency of the signal.

39. An electromagnetic wave in free space has a wavelength of 2.0×10^{-10} m. State the region of the electromagnetic spectrum to which this wave belongs.

40. A displacement-time graph for a particle in a medium shows that the time interval between the first peak and the fourth peak is 0.060 s. Calculate the period of oscillation of the particle.

41. Explain why sound waves travelling through air cannot be polarised.

42. A stationary loudspeaker emits sound of frequency 500 Hz. A sound detector is stationary while a second source emitting sound of the same frequency moves towards the detector at 34 m s^{-1}. The speed of sound in air is 340 m s^{-1}. Calculate the wavelength of the sound waves reaching the detector from the moving source.

43. In a ripple tank, a straight vibrator produces plane water waves. The distance between 5 consecutive wave crests is 16.0 cm. The speed of the water waves is 0.24 m s^{-1}. Calculate the frequency of the vibrator.

44. A point source of light emits radiation uniformly in all directions. At a distance of 2.0 m from the source, the intensity is I_0. Determine the intensity of the light at a distance of 6.0 m from the source in terms of I_0.

45. A beam of plane-polarised light of intensity I_0 passes through two polarising filters in series. The transmission axis of the first filter is parallel to the plane of polarisation of the incident light. The transmission axis of the second filter is at an angle of 30° to the transmission axis of the first filter. Calculate the intensity of the light emerging from the second filter in terms of I_0.

46. A progressive wave travels with a speed of 320 m s^{-1} and has a frequency of 400 Hz. Calculate the minimum distance between two points on the wave that have a phase difference of π rad.

47. A radio signal is transmitted from Earth to a spacecraft at a distance of 7.5×10^8 km. The speed of light in free space is 3.0×10^8 m s^{-1}. Calculate the time taken for the signal to reach the spacecraft.

48. An alternating potential difference of peak value 6.0 V is connected to the Y-input of a cathode-ray oscilloscope (CRO). The Y-gain is set to 1.5 V cm^{-1}. Determine the peak-to-peak height of the trace displayed on the screen.

49. A train sounds a whistle as it moves at a constant speed of 25.0 m s^{-1} directly towards a stationary observer. The observed frequency of the whistle is 440 Hz. The speed of sound in air is 330 m s^{-1}. Calculate the frequency of the whistle emitted by the train.

50. State the approximate range of wavelengths in free space of electromagnetic waves visible to the human eye.

Practice（综合练习）· 25

1. (a) Describe what is meant by wave motion. (b) Use the definitions of speed, frequency and wavelength to derive the wave equation $v = f\lambda$.

2. A sound wave of frequency 400 Hz is displayed on a cathode-ray oscilloscope (CRO) screen. The trace shows exactly 2.5 complete cycles over a horizontal distance of 10 divisions. (a) Calculate the period of the sound wave. (b) Determine the time-base setting of the CRO in ms div^{-1}.

3. A small source emits sound waves uniformly in all directions. The power of the source is 1.5 W. (a) Calculate the intensity of the sound waves at a distance of 4.0 m from the source. (b) The amplitude of the sound wave at 4.0 m is A. Determine the distance from the source where the amplitude is $0.50A$.

4. (a) Compare transverse and longitudinal waves in terms of the oscillations of particles. (b) State one example of a transverse wave and one example of a longitudinal wave.

5. A transverse progressive wave travels along a stretched string. The displacement-distance graph at time $t = 0$ shows a wave with wavelength 0.80 m and amplitude 0.015 m. The wave has a speed of 12 m s^{-1}. (a) Calculate the frequency of the wave. (b) Determine the phase difference between two points on the string separated by a distance of 0.30 m.

6. A longitudinal wave travels through a medium in the positive x-direction. Fig. 6 shows the variation of displacement s of the particles with distance x along the wave at a particular instant. Positive values of s represent displacement in the positive x-direction. (a) Determine the amplitude and wavelength of the wave. (b) State one value of x where a compression is occurring. (c) State one value of x where a rarefaction is occurring.

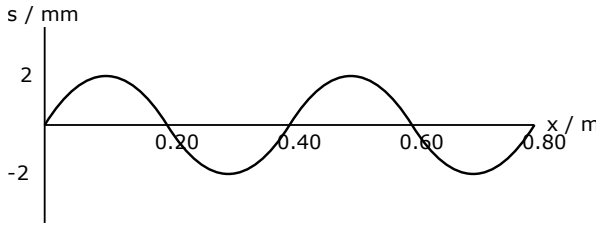

Fig. 6.

7. A car horn emits sound of frequency 450 Hz. The car moves directly towards a stationary observer at a constant speed of 25 m s^{-1}. The speed of sound in the air is 340 m s^{-1}. Calculate the frequency of the sound heard by the observer.

8. A train travels directly away from a stationary observer at a constant speed of 35 m s^{-1}. The train sounds its whistle, which has a frequency of 620 Hz. The speed of sound in the air is 330 m s^{-1}. (a) Calculate the frequency of the sound heard by the observer. (b) State and explain how the observed frequency changes if the train moves away at a higher constant speed.

9. (a) State two properties common to all electromagnetic waves. (b) State the approximate range of wavelengths in free space of X-rays. (c) An electromagnetic wave in free space has a frequency of 4.5×10^{14} Hz. Show that this wave is in the visible region of the electromagnetic spectrum.

10. A plane-polarised electromagnetic wave of intensity 4.2 W m^{-2} is incident normally on a polarising filter. The transmission axis of the filter is at an angle of 35° to the plane of polarisation of the incident wave. (a) State what is meant by polarisation. (b) Calculate the intensity of the transmitted wave.

11. A plane-polarised light beam of intensity I_0 passes through a polarising filter. The filter is rotated by an angle θ about the direction of the beam. The transmitted intensity is measured to be $0.25I_0$. (a) Calculate the possible values of θ between 0° and 90°. (b) Explain why longitudinal waves cannot be polarised.

12. A progressive wave has a frequency of 5.0 kHz and travels at a speed of 1500 m s^{-1}. (a) Calculate the wavelength of the wave. (b) Two points on the path of the wave are separated by a distance of 0.12 m. Calculate the phase difference between the oscillations at these two points.

13. Fig. 13 shows the trace of a sound wave on the screen of a cathode-ray oscilloscope (CRO). The time-base is set to 2.5 ms div^{-1} and the y-gain is set to 5.0 mV div^{-1}. (a) Determine the amplitude of the signal. (b) Determine the frequency of the sound wave.

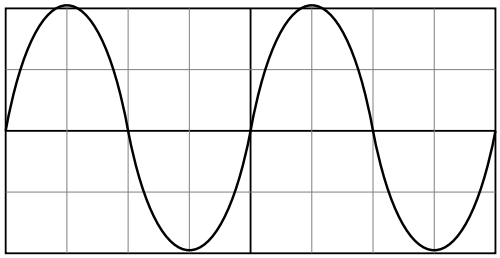

Fig. 13.

14. A progressive wave transfers energy through a medium. At point A, the wave has an amplitude of 4.0 mm and an intensity of 1.6×10^{-4} W m^{-2}. At point B, further along the path of the wave, the intensity has decreased to 9.0×10^{-5} W m^{-2} due to the spreading of the wave. Calculate the amplitude of the wave at point B.

15. An ambulance travels at a constant speed of 28 m s^{-1} along a straight road, away from a stationary pedestrian. The ambulance siren emits a sound of frequency 850 Hz. The speed of sound in the air is 340 m s^{-1}. (a) Calculate the frequency of the sound heard by the pedestrian. (b) The ambulance now travels towards the pedestrian at the same speed. Calculate the change in the observed frequency compared to when it was travelling away.

16. A stationary observer detects sound from a moving source. The source emits sound of frequency 1200 Hz. The observed frequency is 1150 Hz. The speed of sound in air is 330 m s^{-1}. (a) State whether the source is moving towards or away from the observer. (b) Calculate the speed of the source.

17. (a) State the region of the electromagnetic spectrum that includes waves with a wavelength of 2.0×10^{-2} m in free space. (b) Calculate the frequency of these waves. (c) State the approximate range of frequencies in free space for visible light.

18. A plane-polarised electromagnetic wave passes through a polarising filter. The initial amplitude of the wave is A_0. The transmission axis of the filter is at an angle of 50° to the plane of polarisation. (a) State the equation relating the transmitted intensity I to the incident intensity I_0. (b) Calculate the amplitude of the transmitted wave in terms of A_0.

19. A progressive water wave on the surface of a pond has a frequency of 2.5 Hz and a wavelength of 0.16 m. (a) Calculate the speed of the wave. (b) A small leaf floats on the surface of the pond. Describe the motion of the leaf as the wave passes. (c) Explain how this demonstrates that the wave transfers energy but not matter.

20. Fig. 20 shows the traces of a sound wave detected by two microphones separated by a distance of 0.42 m along the direction of wave travel. The time-base of the CRO is 1.2 ms div^{-1}. (a) Determine the frequency of the sound wave. (b) Determine the phase difference between the two waves. (c) Calculate the speed of the sound wave.

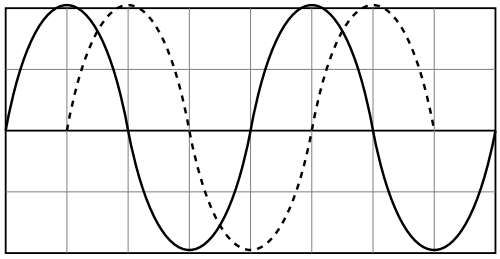

Fig. 20.

21. A police car sounds a siren of frequency 1500 Hz. The car is travelling at 30 m s^{-1} towards a stationary observer. The speed of sound is 340 m s^{-1}. (a) Calculate the observed frequency. (b) The car passes the observer and continues at the same speed. State the new observed frequency. (c) Explain why the observed frequency changes abruptly as the car passes the observer.

22. A transverse wave travels along a string at a speed of 24 m s^{-1}. A point on the string undergoes 150 complete oscillations in 5.0 s. (a) Calculate the period of the oscillations. (b) Calculate the wavelength of the wave. (c) The amplitude of the wave is 3.5 mm. Calculate the total distance moved by a point on the string during one complete oscillation.

23. A laser beam emits plane-polarised light of intensity 12 W m^{-2}. The light passes through a polarising filter. The transmitted intensity is 3.0 W m^{-2}. (a) Calculate the angle between the transmission axis of the filter and the plane of polarisation of the incident light. (b) A second identical filter is placed after the first one, with its transmission axis rotated by a further 30°. Calculate the final intensity.

24. Gamma rays and radio waves are both part of the electromagnetic spectrum. (a) State the approximate range of wavelengths for gamma rays and for radio waves in free space. (b) Calculate the ratio of the frequency of a gamma ray of wavelength 1.0×10^{-12} m to the frequency of a radio wave of wavelength 1.0×10^{3} m.

25. A stationary observer detects sound from a moving train. When the train is approaching the observer at constant speed v_s, the observed frequency is f_1. When the train is moving away from the observer at the same speed v_s, the observed frequency is f_2. The ratio f_1/f_2 is 1.15. The speed of sound in air is 340 m s^{-1}. Determine the speed v_s of the train.

Advanced Practice（培优）· 20

1. (a) Use the definitions of speed v, frequency f and wavelength λ to derive the wave equation $v = f\lambda$. (b) A progressive wave travels along a stretched spring. The distance between an adjacent peak and trough is 0.45 m. The wave transfers energy at a rate of 2.4 W across a cross-sectional area of 3.0×10^{-4} m^2. The time taken for a particle in the spring to complete one full oscillation is 0.12 s. Calculate: (i) the speed of the wave. (ii) the intensity of the wave.

2. A microphone is connected to a cathode-ray oscilloscope (CRO) to detect a sound wave. The time-base setting is 2.5 ms div^{-1} and the y-gain is 5.0 mV div^{-1}. The trace shows a sinusoidal wave where the horizontal distance between a peak and the adjacent trough is 3.2 divisions, and the vertical distance between them is 4.6 divisions. (a) Calculate the frequency of the sound wave. (b) Calculate the amplitude of the signal in volts. (c) The intensity of the sound wave is now doubled. The frequency remains unchanged. Determine the new vertical distance between a peak and the adjacent trough on the CRO trace.

3. (a) Compare the motions of particles in transverse and longitudinal progressive waves. (b) A longitudinal wave travels through a medium with a speed of 340 m s^{-1}. A displacement-distance graph is plotted for the wave at a given instant. The distance between a compression and the nearest rarefaction is 0.85 m. Calculate: (i) the frequency of the wave. (ii) the phase difference between two particles in the medium separated by a distance of 1.19 m.

4. A train travels at a constant speed along a straight track, moving directly towards a stationary observer. The train emits a sound of frequency 450 Hz. The observer hears a sound of frequency 495 Hz. The speed of sound in the air is 330 m s^{-1}. (a) Calculate the speed of the train. (b) The train passes the observer and continues to travel at the same constant speed away from the observer. Calculate the new frequency heard by the observer.

5. An electromagnetic wave is travelling in free space. The wave is produced by a source that oscillates with a period of 1.8×10^{-15} s. (a) State the speed of the wave in free space. (b) Calculate the wavelength of the wave. (c) State the principal region of the electromagnetic spectrum to which this wave belongs. (d) State whether this wave is visible to the human eye. Justify your answer.

6. A plane-polarised electromagnetic wave of intensity I_0 is incident normally on a polarising filter. The transmission axis of the filter is at an angle θ to the plane of polarisation of the incident wave. The transmitted wave has an intensity of $0.15I_0$. (a) Calculate the angle θ. (b) A second identical polarising filter is placed after the first one. The transmission axis of the second filter is at an angle of 90° to the plane of polarisation of the original incident wave. Calculate the intensity of the wave transmitted through the second filter, in terms of I_0.

7. Fig. 7 shows the variation with time t of the displacement x of a particle in a progressive wave. The wave is travelling with a speed of 1.5 km s^{-1}. (a) Determine the amplitude and frequency of the wave. (b) Calculate the wavelength of the wave. (c) Determine the phase difference between two particles in the wave that are separated by a distance of 2.0 m.

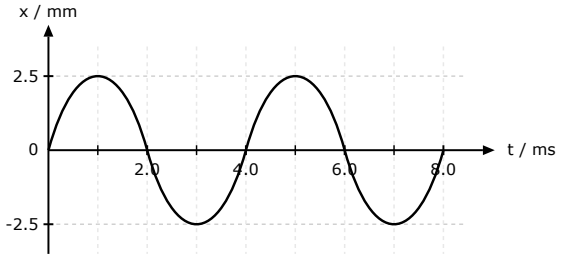

Fig. 7. Displacement-time graph showing the variation of displacement with time for a particle in the progressive wave.

8. A stationary observer stands near a straight road. A car travels along the road at a constant speed v_c, sounding its horn which emits a frequency of 850 Hz. As the car approaches the observer, the observed frequency is 910 Hz. The speed of sound in air is 340 m s^{-1}. (a) Calculate the speed of the car v_c. (b) A second car travels along the same road in the opposite direction, moving away from the observer at a speed of $0.5v_c$. It sounds a horn of the same frequency 850 Hz. Calculate the difference between the frequencies heard by the observer from the two cars.

9. An electromagnetic wave in free space has a wavelength of 3.0×10^{-2} m. (a) State the principal region of the electromagnetic spectrum for this wave. (b) State one property that is common to all electromagnetic waves but is not shared by sound waves. (c) The wave is initially unpolarised. It is passed through a polarising filter, becoming plane-polarised with an amplitude A_0. It then passes through a second polarising filter. The transmission axis of the second filter is rotated by an angle of 45° relative to the first. Calculate the ratio of the final amplitude to the initial amplitude A_0.

10. Two points, P and Q, on a progressive wave are separated by a distance of 0.15 m. The wave has a speed of 2.4 m s^{-1} and a frequency of 4.0 Hz. (a) Calculate the phase difference between the oscillations at P and Q. (b) The intensity of the wave is 1.6×10^{-3} W m^{-2} and its amplitude is 4.2 mm. The amplitude of the wave is then reduced to 1.4 mm. Calculate the new intensity of the wave.

11. A source of sound of frequency f_s moves directly away from a stationary observer at a constant speed v_s. The speed of sound in air is v. (a) Explain why the frequency of the sound heard by the observer is less than f_s. (b) An ambulance moving away from a stationary observer at 25 m s^{-1} emits a siren sound of frequency 1200 Hz. The speed of sound is 340 m s^{-1}. Calculate the wavelength of the sound waves detected by the observer.

12. A transverse progressive wave travels along a string. Fig. 12 shows the displacement-distance graph of the wave at time $t = 0$. The wave has a frequency of 5.0 Hz and travels to the right. (a) Determine the speed of the wave. (b) Point X on the string is at distance $x = 0.40$ m. State the displacement of point X at $t = 0.10$ s.

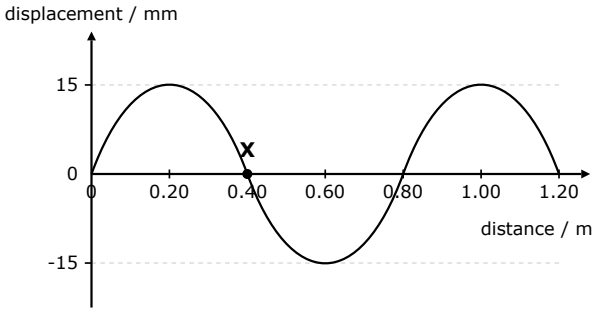

Fig. 12. Displacement-distance graph of a transverse progressive wave along a string at time t = 0, showing point X at distance 0.40 m.

13. A laser beam consists of a progressive electromagnetic wave. The beam has a circular cross-section of diameter 2.4 mm and delivers a power of 5.0 mW. (a) Calculate the intensity of the laser beam. (b) The laser beam is replaced by a second beam of the same power but with a diameter of 1.2 mm. Determine the ratio of the amplitude of the second beam to the amplitude of the first beam.

14. Plane-polarised visible light of intensity I_0 passes through a series of three polarising filters. The transmission axis of the first filter is at an angle of 20° to the plane of polarisation of the incident light. The transmission axis of the second filter is at an angle of 50° to the plane of polarisation of the incident light. The transmission axis of the third filter is at an angle of 90° to the plane of polarisation of the incident light. Calculate the intensity of the light emerging from the third filter, in terms of I_0.

15. An X-ray has a wavelength of 2.5×10^{-10} m in free space. (a) Calculate the frequency of this X-ray. (b) State the approximate range of wavelengths in free space for visible light. (c) The X-ray beam has an intensity of 4.5 W m^{-2}. Calculate the energy transferred by the beam through a cross-sectional area of 2.0×10^{-4} m^2 in a time of 1.0 minute.

16. A buzzer emits a sound of frequency 820 Hz. The buzzer is attached to a trolley that travels at a constant speed along a straight track. A stationary microphone is placed on the track ahead of the trolley. The microphone is connected to a CRO. The time-base of the CRO is set to 0.20 ms div^{-1}. The trace on the CRO shows a sinusoidal wave where one complete cycle occupies exactly 5.8 divisions. The speed of sound in air is 343 m s^{-1}. (a) Determine the frequency of the sound detected by the microphone. (b) Calculate the speed of the trolley.

17. A sound wave travels through water. The wave is longitudinal. The speed of sound in water is 1500 m s^{-1}. The distance between a compression and the adjacent rarefaction is 0.15 m. (a) Calculate the period of the wave. (b) The intensity of the sound wave is I. The wave travels into a different medium where its speed decreases, but its frequency remains constant. The amplitude of the wave in the new medium is half of its original amplitude. State the new intensity in terms of I.

18. Two microphones, M_1 and M_2, are placed in a straight line extending from a stationary loudspeaker emitting a constant frequency sound. The microphones are connected to the two input channels of a CRO. The traces for the two signals are displayed on the same screen. The time-base is set to 0.50 ms div^{-1}. The traces show that the two signals have the same period of 4.0 divisions, but the peaks of the signal from M_2 occur 0.60 divisions later than the peaks from M_1. The speed of sound is 330 m s^{-1}. (a) Calculate the frequency of the sound. (b) Calculate the phase difference, in radians, between the two signals. (c) Calculate the minimum distance between M_1 and M_2.

19. A radio station transmits an electromagnetic wave of frequency 95.0 MHz. (a) Calculate the wavelength of the wave in free space. (b) Estimate the ratio of the wavelength of this radio wave to the wavelength of red visible light.

20. Plane-polarised light of intensity I_0 is incident on a polarising filter. The transmission axis of the filter is initially parallel to the plane of polarisation of the light. The filter is then rotated by an angle θ about the direction of the light beam. The transmitted light has an intensity of $I_0/3$. (a) Calculate the smallest possible value of θ. (b) The filter is rotated further until the transmitted intensity is zero. State the total angle through which the filter has been rotated from its initial position. (c) The initial plane-polarised light has an amplitude of 5.0 V m^{-1}. Calculate the amplitude of the transmitted light when the intensity is $I_0/3$.

8 Superposition（波的叠加）

Principle of superposition, stationary waves, diffraction, two-source interference, and diffraction gratings.

Rules 规则

- The principle of superposition states that when two or more waves meet at a point, the resultant displacement is the algebraic sum of the individual displacements (叠加原理).
- Stationary waves are formed by the superposition of two progressive waves of the same frequency and amplitude travelling in opposite directions; the distance between adjacent nodes or antinodes is $\frac{\lambda}{2}$ (驻波).
- Diffraction is the spreading of a wave as it passes through a gap or around an edge, and is most prominent when the gap width is comparable to the wavelength (衍射).
- Coherence requires two wave sources to have a constant phase difference, which implies they must have the same frequency (相干性).
- For double-slit interference, the fringe spacing x is given by $\lambda = \frac{ax}{D}$, where a is the slit separation and D is the distance to the screen (双缝干涉).
- For a diffraction grating, the maxima occur at angles θ given by $d\sin\theta = n\lambda$, where d is the grating spacing and n is the order of the maximum (衍射光栅).

Worked Examples 例题

Example 8.1

A stationary wave is formed on a stretched string. The distance between the first node and the fourth node is 0.60 m. Determine the wavelength of the progressive waves forming the stationary wave.

Solution

The distance between adjacent nodes in a stationary wave is $\frac{\lambda}{2}$. Between the first node and the fourth node, there are 3 internodal segments. Therefore, the total distance is $3 \times \frac{\lambda}{2} = 0.60$ m. Solving for the wavelength λ: $\frac{3\lambda}{2} = 0.60$, which gives $\lambda = \frac{1.20}{3} = 0.40$ m.

Example 8.2

Light of wavelength 5.9×10^{-7} m is incident normally on a double slit with a separation of 0.30 mm. An interference pattern is observed on a screen placed 2.5 m from the slits. Calculate the separation between adjacent bright fringes.

Solution

Use the double-slit interference equation $\lambda = \frac{ax}{D}$. Rearrange the formula to find the fringe spacing x: $x = \frac{\lambda D}{a}$. Substitute the given values, ensuring all units are in metres ($a = 0.30 \times 10^{-3}$ m): $x = \frac{5.9\times10^{-7}\times2.5}{0.30\times10^{-3}}$. Calculating this gives $x = 4.917 \times 10^{-3}$ m. To two significant figures, the separation is 4.9×10^{-3} m.

Example 8.3

A diffraction grating has 500 lines per mm. Monochromatic light is incident normally on the grating. The second-order maximum is observed at an angle of 32°. Calculate the wavelength of the light.

Solution

First, determine the grating spacing d, which is the distance between adjacent lines. $d = \frac{1}{500}$ mm $= 2.0 \times 10^{-3}$ mm $= 2.0 \times 10^{-6}$ m. Use the diffraction grating equation $d\sin\theta = n\lambda$. Rearrange for wavelength λ: $\lambda = \frac{d\sin\theta}{n}$. Substitute the values for the second order ($n = 2$): $\lambda = \frac{2.0\times10^{-6}\times\sin(32^\circ)}{2}$. Calculating this gives $\lambda = 5.3 \times 10^{-7}$ m.

Example 8.4

Two coherent waves meet at a point in phase. The amplitude of the first wave is 3.0 cm and the amplitude of the second wave is 4.0 cm. State the resultant amplitude at this point and explain your reasoning.

Solution

According to the principle of superposition, the resultant displacement is the algebraic sum of the individual displacements. Since the two waves meet exactly in phase, their maximum positive displacements (amplitudes) occur at the exact same time. Therefore, the resultant amplitude is the direct sum of the individual amplitudes: Resultant amplitude = 3.0 cm + 4.0 cm = 7.0 cm.

考点总览 Key Points

- ☐ 我理解波的叠加原理 (principle of superposition) 并能用于分析波的叠加
- ☐ 我会用图像法解释驻波 (stationary wave) 的形成并准确识别波节 (node) 与波腹 (antinode)
- ☐ 我熟悉利用微波 (microwaves)、弦线 (stretched strings) 和空气柱 (air columns) 产生驻波的实验
- ☐ 我能利用驻波中相邻波节或波腹的位置关系确定波长（相邻间距为 $\frac{\lambda}{2}$）
- ☐ 我理解衍射 (diffraction) 的含义，并掌握缝宽与波长相对大小对衍射效果的影响
- ☐ 我理解相干性 (coherence) 与干涉 (interference) 的概念及观察到清晰干涉条纹的条件
- ☐ 我熟悉在水波槽 (ripple tank)、声波、光波及微波中演示双波源干涉的实验
- ☐ 我掌握双缝干涉公式 $\lambda = \frac{ax}{D}$ 并能熟练进行相关物理量的计算
- ☐ 我掌握衍射光栅 (diffraction grating) 方程 $d\sin\theta = n\lambda$ 并能进行定量计算
- ☐ 我能描述使用衍射光栅测定光波波长的实验方法与原理

Basic Drill（入门综合）· 50

1. State the principle of superposition.

2. A stationary wave is formed on a stretched string. The distance between two adjacent nodes is 0.25 m. Calculate the wavelength of the progressive waves forming the stationary wave.

3. A transmitter emits microwaves towards a metal reflector, setting up a stationary wave between the transmitter and the reflector. A probe detector moved along the line between them detects adjacent intensity maxima separated by a distance of 1.5 cm. The speed of microwaves is 3.0×10^8 m s^{-1}. Calculate the frequency of the microwaves.

4. Define a node on a stationary wave.

5. A stationary sound wave is set up in a tube of length 0.18 m that is closed at one end and open at the other. The wave is in its fundamental mode (lowest frequency), with a node at the closed end and an antinode at the open end. Calculate the wavelength of the sound wave.

6. State the phase difference between the oscillations of any two particles situated between two adjacent nodes on a stationary wave.

7. A stationary wave with three loops (antinodes) is established on a wire of length 1.20 m fixed at both ends. Determine the distance between adjacent nodes.

8. Explain what is meant by the term diffraction.

9. Plane water waves in a ripple tank pass through a narrow gap. State the condition for the diffraction of the waves to be most significant.

10. Plane water waves of wavelength 2.5 cm are incident on a barrier with a gap of width 2.5 cm. The gap width is then increased to 15.0 cm. State the effect of this increase in gap width on the amount of diffraction observed.

11. Explain why sound waves diffract significantly around an open doorway whereas visible light waves do not.

12. Explain what is meant by coherent wave sources.

13. Two coherent sources, S_1 and S_2, emit waves of wavelength λ in phase. State the condition for constructive interference to occur at a point P in terms of the path difference ($S_2\mathrm{P} - S_1\mathrm{P}$) and wavelength λ.

14. Two coherent sources emit waves of wavelength 4.0 cm in phase. At a certain point, the distance from one source is 28.0 cm and from the other source is 34.0 cm. State whether constructive or destructive interference occurs at this point.

15. In a double-slit experiment using monochromatic light, the slit separation is 0.25 mm and the screen is placed at a distance of 2.0 m from the slits. The observed fringe separation is 4.8 mm. Calculate the wavelength of the light.

16. In a double-slit interference experiment, the distance D between the slits and the screen is doubled, while the slit separation a is halved. State the factor by which the fringe separation x changes.

17. In a double-slit experiment using monochromatic light of wavelength 500 nm, the central bright fringe is formed at point O. Point Q is the centre of the first dark fringe adjacent to O. State the path difference between the waves arriving at Q from the two slits.

18. In a double-slit experiment, blue light is replaced with red light without changing the slit separation or screen distance. State and explain the change, if any, in the fringe separation.

19. A diffraction grating has 500 lines per mm. Calculate the grating spacing d in metres.

20. Monochromatic light of wavelength 600 nm is incident normally on a diffraction grating with line spacing $d = 2.00 \times 10^{-6}$ m. Calculate the angle θ between the central maximum and the first-order maximum.

21. Light of wavelength 532 nm is incident normally on a diffraction grating with line spacing $d = 1.60 \times 10^{-6}$ m. Determine the highest order of diffraction maximum, $n_{\max}$, that can be observed.

22. A beam of monochromatic light is incident normally on a diffraction grating having 4.00×10^5 lines m^{-1}. The second-order diffraction maximum is observed at an angle of 28.0° to the straight-through direction. Calculate the wavelength of the light. Give your answer to 3 significant figures.

23. A parallel beam of white light is incident normally on a diffraction grating. State whether red light or violet light is diffracted through the larger angle in the first-order spectrum.

24. In the diffraction grating equation $d \sin\theta = n\lambda$, state what the integer n represents.

25. State one advantage of using a diffraction grating rather than a double slit to determine the wavelength of monochromatic light.

26. Two coherent waves, each having an amplitude of 2.5 mm, arrive at a point in phase. Calculate the resultant amplitude at this point.

27. A stationary sound wave is formed in a tube. The distance between two adjacent nodes is 18 cm. Calculate the wavelength of the sound wave.

28. A microwave transmitter emits waves towards a metal reflecting plate to set up a stationary wave. A detector moved along the line between the transmitter and the plate detects adjacent positions of minimum intensity separated by 1.5 cm. Determine the wavelength of the microwaves.

29. A stationary wave on a stretched string of length 1.20 m has fixed ends and vibrates with three antinodal loops. Calculate the wavelength of the wave.

30. A stationary sound wave is formed in a tube of length 0.25 m that is open at one end and closed at the other. The tube is vibrating in its fundamental mode. Neglecting end corrections, calculate the wavelength of the sound wave.

31. State two conditions required for two progressive waves to produce a stationary wave when they meet.

32. State the difference between a node and an antinode in terms of the amplitude of vibration.

33. Explain the meaning of the term diffraction.

34. Plane water waves in a ripple tank are incident normally on a gap in a barrier. Describe the effect on the diffraction of the waves when the gap width is reduced so that it is equal to the wavelength of the waves.

35. Plane water waves pass through a gap in a barrier in a ripple tank. State the condition relating the gap width to the wavelength of the waves for circular wavefronts to be produced after passing through the gap.

36. Explain what is meant by two wave sources being coherent.

37. Two coherent sources emit waves of wavelength 4.0 cm in phase. At a point P, the path difference between the waves from the two sources is 12.0 cm. State the type of interference that occurs at P.

38. Two coherent sources emit light in phase with a wavelength of 600 nm. State the minimum non-zero path difference between the waves arriving at a point on a screen that produces completely destructive interference.

39. In a double-slit experiment, monochromatic light passes through two slits separated by 0.25 mm. The interference fringes are viewed on a screen placed 1.80 m from the slits. The distance across 5 consecutive bright fringes is 18.4 mm. Calculate the wavelength of the light.

40. Light of wavelength 589 nm is incident on a pair of parallel slits separated by 0.30 mm. A screen is placed at a distance of 1.50 m from the slits. Calculate the fringe spacing on the screen.

41. Two loudspeakers connected to the same signal generator emit sound waves of frequency 680 Hz in phase. The speed of sound in air is 340 m s^{-1}. Calculate the path difference from the two speakers to the first position of minimum sound intensity.

42. In a two-source interference experiment with microwaves of wavelength 2.8 cm, the sources oscillate in phase. Determine the path difference from the two sources to the second bright maximum on one side of the central maximum.

43. In a Young's double-slit experiment using white light, describe the appearance of the central fringe.

44. A diffraction grating has 500 lines per millimetre. Calculate the line spacing d of the grating in metres.

45. Monochromatic light of wavelength 500 nm is incident normally on a diffraction grating. The third-order diffraction maximum is observed at an angle of 30.0° to the central maximum. Calculate the line spacing d of the grating.

46. Light is incident normally on a diffraction grating having 600 lines per millimetre. The first-order diffraction maximum is observed at an angle of 21.5°. Calculate the wavelength of the light.

47. A diffraction grating has a line spacing of 1.80 μm. Monochromatic light of wavelength 480 nm is incident normally on the grating. Calculate the total number of diffraction maxima that can be observed.

48. Light consisting of two wavelengths, $\lambda_1 = 400$ nm and $\lambda_2 = 700$ nm, is incident normally on a diffraction grating of line spacing 2.50 μm. Calculate the angular separation between the first-order maxima of the two wavelengths.

49. A stationary sound wave is established in an open-ended organ pipe. The distance between two consecutive antinodes is 0.42 m. Determine the wavelength of the sound wave.

50. Laser light of wavelength 633 nm is incident on a double slit. The interference pattern is observed on a screen 2.20 m away, where the fringe spacing is measured to be 3.6 mm. Calculate the separation of the two slits.

Practice（综合练习）· 25

1. A microwave transmitter directs waves of frequency 1.20×10^{10} Hz towards a flat metal reflector. A stationary wave is formed between the transmitter and the reflector. A small microwave receiver is moved along the line joining them. (a) Calculate the distance between adjacent intensity minima detected by the receiver. (b) The receiver is positioned at an intensity minimum. The metal reflector is now replaced by a block of wood that absorbs a significant proportion of the incident microwaves. State and explain the effect on the intensity detected by the receiver.

2. Two pulses on a stretched string approach each other. At a particular instant, they completely overlap. Pulse A has a displacement of +3.5 cm and pulse B has a displacement of −1.2 cm. Calculate the resultant displacement of the string at this instant.

3. A string of length 1.20 m is fixed at both ends. It is vibrated to form a stationary wave with exactly 3 antinodes. (a) State the phase difference between two particles located in adjacent loops of the stationary wave. (b) Calculate the wavelength of the progressive waves forming this stationary wave.

4. A microwave transmitter directs waves towards a flat metal plate. A detector is moved along the line between the transmitter and the plate. A series of maximum and minimum intensity signals is detected. Adjacent minima are separated by a distance of 1.4 cm. (a) Explain how the stationary wave is formed in this experiment. (b) Calculate the wavelength of the microwaves.

5. A glass tube is closed at one end by a movable water surface. A tuning fork is sounded above the open end. The water level is gradually lowered until the first loud sound (fundamental mode) is heard. The length of the air column is 16.2 cm. (a) Identify the positions of the node and antinode in the air column. (b) Calculate the wavelength of the sound wave. (c) Calculate the length of the air column when the next loud sound (the next harmonic) is heard.

6. A stationary wave on a string at time $t = 0$ is shown in Fig. 6. The length of one loop is 0.40 m. (a) Determine the wavelength of the wave. (b) The period of the wave is T. State the displacement of a particle at an antinode at time $t = T/4$.

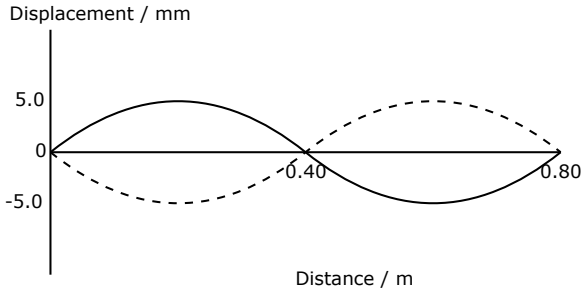

Fig. 6.

7. An organ pipe is open at both ends. The length of the pipe is 0.65 m. A stationary wave is formed in the pipe at its fundamental frequency. (a) State the number of nodes and antinodes in this stationary wave. (b) Calculate the wavelength of the sound wave.

8. (a) Explain the meaning of the term diffraction. (b) Water waves of wavelength 2.5 cm approach a gap of width d in a ripple tank. Describe the qualitative effect on the diffracted waves if d is reduced from 10.0 cm to 2.5 cm.

9. Sound waves of wavelength 0.40 m travel towards a doorway of width 0.90 m. State and explain whether significant diffraction occurs as the sound passes through the doorway.

10. Explain why diffraction is easily observed for sound waves passing through an open window, but is not easily observed for light waves passing through the same window.

11. Microwaves of wavelength 3.0 cm are directed at a metal screen containing a slit of width 4.0 cm. A microwave detector is moved in a wide arc on the other side of the screen. (a) State what is observed by the detector. (b) The slit width is increased to 12.0 cm. Describe the change in the observation.

12. (a) Define coherence. (b) Explain why two separate incandescent light bulbs cannot produce an observable interference pattern.

13. Two dippers in a ripple tank oscillate in phase to produce coherent water waves of wavelength 1.5 cm. Point P on the water surface is 12.0 cm from one dipper and 16.5 cm from the other. (a) Calculate the path difference at P. (b) State and explain the type of interference occurring at P.

14. Two loudspeakers, S_1 and S_2, are connected to the same signal generator. A microphone is moved slowly along a line parallel to the speakers. A series of maxima and minima in sound intensity is detected. (a) Explain how the minima are formed. (b) State what happens to the distance between adjacent minima if the frequency of the sound is increased.

15. Light of wavelength 589 nm is incident normally on a double slit with a separation of 0.25 mm. An interference pattern is observed on a screen at a distance of 1.8 m, as shown in Fig. 15. Calculate the fringe separation on the screen.

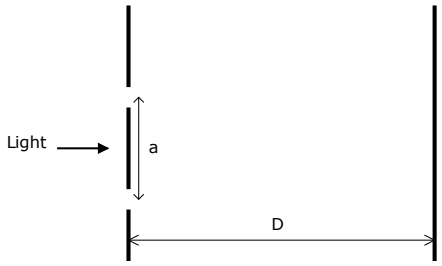

Fig. 15. (not to scale)

16. In a double-slit experiment using light of wavelength 650 nm, the distance across 5 adjacent bright fringes (from the centre of the 1st to the centre of the 5th) is 1.4 cm. The screen is 2.4 m away from the slits. (a) Calculate the fringe separation x. (b) Calculate the slit separation a.

17. A double-slit interference pattern is formed on a screen. State and explain the effect on the fringe separation if: (a) the screen is moved further away from the slits; (b) the slit separation is increased.

18. State three conditions that must be met for two light sources to produce an observable two-source interference pattern.

19. Two microwave transmitters, A and B, emit coherent waves of wavelength 2.8 cm in phase. A detector is placed at point Q. The distance AQ is 45.0 cm and BQ is 54.8 cm. Determine the phase difference between the waves arriving at Q.

20. Light of wavelength 450 nm is incident normally on a diffraction grating. The second-order maximum is observed at an angle of 32°. Calculate the slit spacing d of the grating.

21. A diffraction grating has 600 lines per millimetre. Light of wavelength 520 nm is incident normally on the grating. Calculate the angle of the first-order maximum.

22. A laser beam of wavelength 633 nm is directed normally at a diffraction grating with a slit spacing $d = 2.5 \times 10^{-6}$ m. Determine the maximum number of orders that can be observed on one side of the central maximum.

23. Two coherent point sources, S_1 and S_2, emit sound waves in phase. The speed of sound in the air is 330 m s^{-1} and the initial frequency of the sources is 220 Hz. A microphone is placed at point P, which is 4.00 m from S_1 and 6.40 m from S_2. (a) Calculate the path difference at P in terms of the initial wavelength λ. (b) Determine the phase difference between the waves arriving at P from the two sources. (c) The frequency of the sources is gradually increased. Calculate the lowest new frequency at which the sound intensity at P becomes a maximum.

24. State two reasons why a diffraction grating is usually preferred over a double slit for determining the wavelength of light.

25. A diffraction grating with 500 lines per mm is placed 1.50 m from a flat screen. The light is incident normally. The distance on the screen from the central maximum to the first-order maximum is 0.45 m. Calculate the wavelength of the incident light.

Advanced Practice（培优）· 20

1. A microwave transmitter is directed at a flat metal plate. A detector is moved along the line between the transmitter and the plate. Maxima and minima of intensity are detected. The speed of microwaves in air is 3.0×10^8 m s^{-1}. (a) Explain how the stationary wave is formed in this region. (b) The detector is moved a distance of 14.4 cm and passes through 9 consecutive minima. Calculate the frequency of the microwaves.

2. Two coherent sound sources S_1 and S_2 are placed 4.0 m apart. A microphone is placed at point P, which is 3.0 m from S_1 and 5.0 m from S_2. The frequency of the sound is 170 Hz and the speed of sound in air is 340 m s^{-1}. (a) Determine the path difference at P in terms of the wavelength λ. (b) State and explain whether P is a position of maximum or minimum intensity.

3. A tuning fork of frequency f is held over a vertical tube containing water. The water level is slowly lowered. The first loud sound is heard when the air column is 0.17 m long. The second loud sound is heard when the air column is 0.51 m long. The speed of sound in air is 340 m s^{-1}. (a) Explain why a loud sound is heard at specific lengths of the air column. (b) Calculate the frequency f.

4. A beam of monochromatic light of wavelength 589 nm is incident normally on a diffraction grating with 600 lines per mm. (a) Calculate the angle of the second-order maximum. (b) Determine the highest order of maximum that can be observed.

5. Light of wavelength 633 nm is incident normally on a double slit. The slit separation is 0.25 mm and the interference pattern is observed on a screen 4.5 m away. (a) State the conditions required for observable two-source interference fringes. (b) Calculate the distance between the central bright fringe and the fifth dark fringe.

6. A horizontal glass tube containing a small amount of fine powder is closed at one flat end. A loudspeaker connected to a signal generator is placed near the open end. A stationary sound wave is formed in the tube, causing the powder to gather into small heaps.
   At a certain frequency, 5 distinct heaps of powder are formed. The distance between the centre of the first heap and the centre of the fifth heap is 24.0 cm.
   (a) Explain how the incident sound wave and the reflected sound wave produce the stationary wave in the tube. (b) The heaps of powder form at the nodes of the stationary wave. Calculate the wavelength of the sound wave. (c) State the phase difference between the oscillations of the air particles located midway between the first and second heaps, and the air particles located midway between the second and third heaps. (d) The frequency of the sound is increased so that the wavelength becomes 8.0 cm. Calculate the new distance between the first heap and the fifth heap.

7. Light containing two distinct wavelengths, 450 nm and 600 nm, is incident normally on a diffraction grating. The grating has 5.0×10^5 lines per metre. Calculate the angular separation between the second-order maxima of these two wavelengths.

8. Microwaves of frequency 1.2×10^{10} Hz are incident on two slits S_1 and S_2 which are 8.0 cm apart. A detector is moved along a line parallel to the slits and 1.5 m away. The speed of microwaves is 3.0×10^8 m s^{-1}. Calculate the distance between adjacent maxima of the detected signal.

9. Water waves in a ripple tank pass through a gap in a barrier. The wavelength of the waves is 1.5 cm. The gap width is 2.0 cm. (a) Explain the meaning of the term diffraction. (b) The gap width is increased to 6.0 cm while the wavelength remains 1.5 cm. Describe the qualitative effect on the diffracted waves.

10. Two triangular pulses A and B are travelling in opposite directions on a stretched string, as shown in Fig. 10. Both pulses have a peak displacement of 3.0 cm and a width of 10.0 cm. The pulses are approaching each other at a speed of 2.0 m s^{-1}. (a) State the principle of superposition. (b) The peaks of the two pulses are initially separated by 30.0 cm. Calculate the time taken for the two peaks to coincide. (c) Describe the resultant shape of the string at the instant the peaks coincide.



Fig. 10.

11. (a) Define coherence. (b) Two independent light bulbs are placed side by side. Explain why they do not produce an observable interference pattern on a screen.

12. A laser beam of wavelength 532 nm is incident normally on a diffraction grating. The first-order maximum is observed at an angle of 15.0°. (a) Calculate the number of lines per millimetre of the grating. (b) The laser is replaced by one emitting light of wavelength 650 nm. State and explain whether the third-order maximum can be observed for this new wavelength.

13. A pipe of length L is open at one end and closed at the other. A stationary sound wave is formed in the pipe at its fundamental frequency f_1. (a) State the positions of the nodes and antinodes at the ends of the pipe. (b) The same pipe is now opened at both ends. Determine the new fundamental frequency f_2 in terms of f_1.

14. White light is incident on a double slit. The central maximum of the interference pattern is white, but the first-order maximum is a spectrum of colours. (a) Explain why the central maximum is white. (b) State which colour, red or blue, is closest to the central maximum in the first-order spectrum, and explain your reasoning.

15. Light from a glowing gas is analysed using a diffraction grating with 5.0×10^5 lines per metre. Two distinct spectral lines are observed in the second order at angles of 23.6° and 30.0°. Calculate the difference in wavelength between these two spectral lines.

16. A stationary wave on a string has a frequency of 20 Hz. The distance between an adjacent node and antinode is 15 cm. (a) Calculate the wavelength of the progressive waves forming this stationary wave. (b) Two points on the string are at distances of 10 cm and 20 cm from the same node. Calculate the phase difference between these two points.

17. Two coherent sources S_1 and S_2 emit microwaves of wavelength 3.0 cm in phase. A detector is placed at point Q. The distance S_1Q is 45.0 cm and S_2Q is 51.0 cm. (a) Calculate the path difference at Q. (b) State whether constructive or destructive interference occurs at Q, and explain your answer.

18. In a double-slit experiment using monochromatic light, fringes are observed on a screen. State and explain the effect on the fringe spacing x if: (a) the distance D to the screen is increased; (b) the slit separation a is decreased.

19. A diffraction grating has N lines per metre. When monochromatic light of wavelength λ is incident normally, the fourth-order maximum is observed, but the fifth-order maximum is not. Formulate an inequality for the slit spacing d in terms of λ.

20. A parallel beam of light containing two distinct wavelengths, λ_1 and λ_2, is incident normally on a diffraction grating. The grating has a line spacing of 2.50 μm.
The second-order diffraction maximum for λ_1 occurs at exactly the same angle θ as the third-order diffraction maximum for λ_2. The wavelength λ_1 is 630 nm.
(a) State what is meant by the term diffraction. (b) Calculate the wavelength λ_2. (c) Calculate the angle θ. (d) Determine the highest order of diffraction maximum that can be observed for the wavelength λ_1. (e) Explain why, for any diffraction grating, the central maximum (zero order) for all wavelengths occurs at the same angle.

9 Electricity（电学）

Fundamental concepts of electric current, potential difference, resistance, and resistivity.

Rules 规则

- Electric current is the rate of flow of charge carriers (电荷载流子). The charge is quantised, meaning it is an integer multiple of the elementary charge e.
- Use $Q = It$ to relate charge (电荷量), current (电流), and time. For a current-carrying conductor, use $I = Anvq$, where n is the number density (数密度) of charge carriers and v is the drift velocity (漂移速度).
- Potential difference (电势差) is defined as the energy transferred per unit charge, $V = \frac{W}{Q}$.
- Electrical power (电功率) can be calculated using $P = VI$, $P = I^2R$, or $P = \frac{V^2}{R}$.
- Resistance (电阻) is defined as the ratio of potential difference to current, $R = \frac{V}{I}$. Ohm's law (欧姆定律) states that current is directly proportional to potential difference for a metallic conductor at constant temperature.
- Resistivity (电阻率) is a property of the material, related to resistance by $R = \frac{\rho L}{A}$.
- The resistance of a filament lamp (灯丝灯泡) increases with current due to temperature rise. The resistance of a thermistor (热敏电阻) decreases as temperature increases, and a light-dependent resistor or LDR (光敏电阻) decreases in resistance as light intensity increases.

Worked Examples 例题

Example 9.1

A copper wire of diameter 1.2 mm carries a current of 5.0 A. The number density of free electrons in copper is 8.5×10^{28} m^{-3}. Calculate the drift velocity of the electrons.

Solution

First, calculate the cross-sectional area $A = \frac{\pi d^2}{4} = \frac{\pi \times (1.2 \times 10^{-3})^2}{4} = 1.13 \times 10^{-6}$ m^2. Using the equation $I = Anvq$, rearrange to make drift velocity v the subject: $v = \frac{I}{Anq}$. Substitute the known values, where $q = e = 1.60 \times 10^{-19}$ C: $v = \frac{5.0}{1.13 \times 10^{-6} \times 8.5 \times 10^{28} \times 1.60 \times 10^{-19}} = 3.3 \times 10^{-4}$ m s^{-1}.

Example 9.2

A nichrome wire has a resistivity of 1.1×10^{-6} Ω m and a cross-sectional area of 4.5×10^{-7} m^2. Determine the length of wire required to produce a resistance of 15 Ω.

Solution

Using the resistivity formula $R = \frac{\rho L}{A}$, rearrange to make length L the subject: $L = \frac{RA}{\rho}$. Substitute the given values: $L = \frac{15 \times 4.5 \times 10^{-7}}{1.1 \times 10^{-6}} = 6.1$ m.

Example 9.3

An electrical component has a potential difference of 12 V across it and a resistance of 8.0 Ω. Calculate the energy transferred by the component in 2.0 minutes.

Solution

First, calculate the power dissipated using $P = \frac{V^2}{R}$. $P = \frac{12^2}{8.0} = 18$ W. Convert the time into seconds: $t = 2.0 \times 60 = 120$ s. The energy transferred is $W = Pt = 18 \times 120 = 2160$ J (or 2.2 kJ to two significant figures).

Example 9.4

Explain why the resistance of a filament lamp increases as the current through it increases.

Solution

As the current increases, the rate of energy dissipation increases, causing the temperature of the filament to rise. This higher temperature causes the metal ions in the lattice to vibrate with greater amplitude. Consequently, the frequency of collisions between the flowing charge carriers (electrons) and the lattice ions increases, which restricts the flow of current and therefore increases the resistance.

考点总览 Key Points

- ☐ 我理解电流 (electric current) 是载流子的定向移动且电荷量是量子化的 (quantised)，并能运用 $Q = It$。
- ☐ 我掌握利用微观公式 $I = Anvq$ 进行电流计算，并理解载流子数密度 (number density) 等物理量。
- ☐ 我能定义电势差 (potential difference) 为单位电荷转移的能量，并能运用公式 $V = \frac{W}{Q}$。
- ☐ 我熟练掌握电功率 (electric power) 的计算，能灵活运用 $P = VI$、$P = I^2R$ 和 $P = \frac{V^2}{R}$。
- ☐ 我能表述欧姆定律 (Ohm's law)，并掌握电阻 (resistance) 的定义及其公式 $V = IR$。
- ☐ 我掌握电阻率 (resistivity) 的概念，并能熟练运用公式 $R = \frac{\rho L}{A}$ 进行计算。
- ☐ 我会绘制恒温金属导体、半导体二极管 (semiconductor diode) 和白炽灯泡的 I–V 特性曲线。
- ☐ 我能解释白炽灯泡 (filament lamp) 随电流增大、温度升高而导致其电阻增大的原因。
- ☐ 我理解光敏电阻 (LDR) 阻值随光照增强而减小，以及热敏电阻 (thermistor) 阻值随温度升高而减小。

Basic Drill（入门综合）· 50

1. A steady current of 0.45 A exists in a wire for a time of 3.0 minutes. Calculate the total charge that passes a cross-section of the wire in this time.

2. A beam of electrons carries a total charge of 7.2 nC. The elementary charge is $e = 1.6 \times 10^{-19}$ C. Calculate the number of electrons in the beam.

3. State the term that describes the property that electric charge only exists in discrete integer multiples of the elementary charge e.

4. A copper wire of cross-sectional area 1.5×10^{-6} m^2 carries a current of 2.4 A. The number density of free electrons in copper is 8.5×10^{28} m^{-3}. Calculate the drift velocity of the electrons. Give your answer to 2 significant figures.

5. Two wires, X and Y, are made of the same metal and connected in series so that they carry the same current. The diameter of wire X is twice the diameter of wire Y. Determine the ratio $\frac{\text{drift velocity in wire X}}{\text{drift velocity in wire Y}}$.

6. Define potential difference across an electrical component.

7. A potential difference of 12 V is applied across a heating element. A charge of 350 C passes through the element. Calculate the energy transferred to the element.

8. An electric motor operates at a potential difference of 24 V with an operating current of 1.5 A. Calculate the electrical power input to the motor.

9. A resistor of resistance 45 Ω carries a current of 0.80 A. Calculate the power dissipated in the resistor.

10. A car headlamp bulb is rated at 12 V, 55 W. Calculate the resistance of the bulb filament when operating at its rated voltage. Give your answer to 2 significant figures.

11. An electric heater of power 1.8 kW is switched on for 45 minutes. Calculate the total energy supplied to the heater in megajoules (MJ).

12. Define the electrical resistance of a conductor.

13. State Ohm's law.

14. When a potential difference of 6.0 V is applied across a component, the current in the component is 15 mA. Calculate the resistance of the component.

15. Explain why the resistance of a filament lamp increases as the current in the lamp increases.

16. The graph shows the I–V characteristic of a semiconductor diode. State the approximate value of the forward threshold potential difference at which the diode begins to conduct significantly.

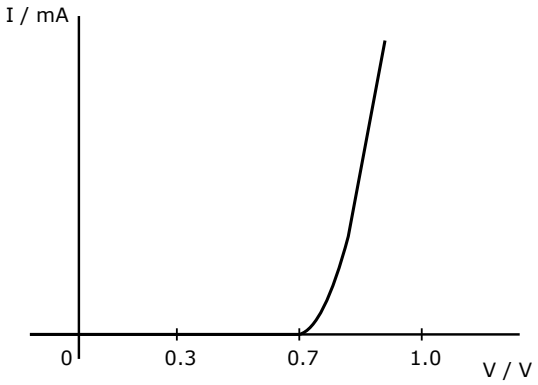

Fig. 16. I-V characteristic of a semiconductor diode

17. A uniform resistance wire has length 2.5 m and cross-sectional area 4.0×10^{-7} m^2. The resistivity of the metal is 1.1×10^{-6} Ω m. Calculate the resistance of the wire.

18. A wire of circular cross-section has length 1.20 m and diameter 0.50 mm. The resistance of the wire is 2.8 Ω. Calculate the resistivity of the material of the wire. Give your answer to 2 significant figures.

19. A cylindrical metal wire has resistance R. The wire is uniformly stretched at constant temperature until its length is tripled, while keeping its total volume constant. Determine the new resistance of the wire in terms of R.

20. State how the resistance of a light-dependent resistor (LDR) changes when the intensity of light incident upon it increases.

21. State how the resistance of a negative temperature coefficient (NTC) thermistor changes when its temperature decreases.

22. The graph shows the I–V characteristic of a metallic conductor at constant temperature. The potential difference across the conductor is 10.0 V when the current is 2.5 A. Calculate the resistance of the conductor.

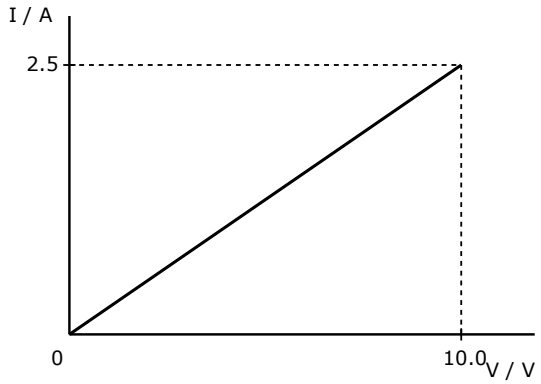

Fig. 22. I-V characteristic of a metallic conductor at constant temperature

23. A wire of cross-sectional area 8.0×10^{-7} m^2 carries a current of 3.2 A. The mean drift velocity of the free electrons is 2.5×10^{-4} m s^{-1}. Calculate the number density of the free electrons in the wire.

24. A dust particle acquires an electrostatic charge. According to the principle of charge quantisation, determine which of the following measured charges is physically impossible: 3.2×10^{-19} C, 4.8×10^{-19} C, 5.6×10^{-19} C, or 6.4×10^{-19} C.

25. In an electric circuit, 150 J of energy is transferred as a charge of 25 C passes through a resistor. Calculate the potential difference across the resistor.

26. A steady current of 0.80 A flows through an electric component for 45 s. The elementary charge is $e = 1.6 \times 10^{-19}$ C. Calculate the number of electrons passing through the component in this time.

27. A cylindrical metal wire of length 2.5 m has a radius of 0.40 mm. The resistance of the wire is 0.85 Ω. Calculate the resistivity of the metal.

28. A heating element of resistance 48 Ω is connected to a 240 V power supply. Calculate the electrical energy converted into thermal energy by the heater in 5.0 minutes.

29. A portable electronic device draws a steady current of 80 mA. Calculate the time, in hours, for a total charge of 576 C to pass through the device.

30. A copper wire of cross-sectional area 1.5×10^{-6} m^2 carries a current of 3.2 A. The number density of free electrons in copper is 8.5×10^{28} m^{-3} and the elementary charge is 1.6×10^{-19} C. Calculate the average drift speed of the free electrons in the wire.

31. A resistor of resistance 25 Ω dissipates a power of 36 W. Calculate the current in the resistor.

32. A cylindrical wire of length L and diameter d has resistance R. A second wire made of the same material has length $3L$ and diameter $2d$. Determine the resistance of the second wire in terms of R.

33. A heating coil is made from a wire of resistivity 1.0×10^{-6} Ω m and cross-sectional area 2.5×10^{-7} m^2. The resistance of the coil is 40 Ω. Calculate the length of the wire.

34. A beam of singly-charged positive ions delivers a total charge of 72 mC to a target in a time of 1.5 minutes. Calculate the mean current of the ion beam.

35. A total charge of 450 C passes through an electrical component in 3.0 minutes when the potential difference across it is 12 V. Calculate the electrical power dissipated in the component.

36. State Ohm's law.

37. An electron beam delivers 3.0×10^{15} electrons to a collector plate every 2.0 s. The elementary charge is $e = 1.6 \times 10^{-19}$ C. Calculate the electric current received by the plate.

38. Two uniform wires, P and Q, are made of the same material and carry the same electric current. The diameter of wire P is twice the diameter of wire Q. State the ratio $\frac{\text{drift speed in P}}{\text{drift speed in Q}}$.

39. A power source transfers 54 J of energy in moving a charge of 4.5 C through a component. Calculate the potential difference across the component.

40. When the potential difference across a filament lamp is 6.0 V, the current is 0.50 A. When the potential difference is increased to 12.0 V, the current is 0.80 A. Calculate the change in the resistance of the lamp.

41. State what is meant by the statement that electric charge is quantised.

42. A uniform metal strip of length 0.80 m has a rectangular cross-section with width 5.0 mm and thickness 0.20 mm. The resistance between its ends is 0.16 Ω. Calculate the resistivity of the metal.

43. A resistor of resistance 150 Ω has a maximum power rating of 6.0 W. Calculate the maximum potential difference that can be safely applied across the resistor.

44. Describe the current in an ideal semiconductor diode when it is reverse biased with a small potential difference.

45. State the name of the charge carriers responsible for electric current in a metallic conductor.

46. An electric kettle rated at 2.2 kW is connected to a 230 V mains supply. Calculate the operating current of the kettle.

47. Define electrical resistance.

48. A metal wire of diameter 1.2 mm carries a current of 2.0 A. The number density of free electrons is 6.0×10^{28} m^{-3} and $e = 1.6 \times 10^{-19}$ C. Calculate the average drift speed of the electrons.

49. State the condition required for the current – potential difference (I–V) graph of a metallic conductor to be a straight line through the origin.

50. A current of 0.25 A flows through a 330 Ω resistor for a time of 40 s. Calculate the energy transferred in the resistor.

Practice（综合练习）· 25

1. (a) Understand that the charge on charge carriers is quantised. State what is meant by the quantisation of charge. (b) A constant electric current of 3.2 A flows through a wire for 5.0 minutes. (i) Calculate the total charge that passes a point in the wire. (ii) Calculate the number of electrons that pass this point.

2. A copper wire has a diameter of 1.6 mm and carries a current of 8.0 A. The number density of free electrons in copper is 8.5×10^{28} m^{-3}. (a) Calculate the cross-sectional area of the wire. (b) Calculate the average drift speed of the free electrons in the wire.

3. (a) Define the potential difference across a component. (b) An electrical component transfers 640 J of energy when a charge of 40 C passes through it. (i) Calculate the potential difference across the component. (ii) The current in the component is 2.5 A. Calculate the time taken for this charge to flow.

4. (a) State Ohm's law. (b) A metallic conductor obeys Ohm's law. When the potential difference across the conductor is 9.0 V, the current is 0.60 A. (i) Calculate the resistance of the conductor. (ii) Calculate the power dissipated in the conductor.

5. A uniform wire of an alloy has a length of 4.5 m and a diameter of 0.80 mm. A potential difference of 12 V is applied across the ends of the wire. The resistivity of the alloy is 1.4×10^{-6} Ω m. (a) Calculate the resistance of the wire. (b) Calculate the power dissipated in the wire. (c) The number density of free electrons in the alloy is 4.0×10^{28} m^{-3}. Calculate the average drift velocity of the free electrons in the wire.

6. A filament lamp is rated at 24 V, 48 W. (a) Calculate the current in the lamp when it operates at its rated voltage. (b) Calculate the resistance of the lamp at this operating voltage. (c) Explain why the resistance of the filament lamp increases as the current increases.

7. Fig. 7 shows the $I - V$ characteristic for a semiconductor component. (a) State the name of this component. (b) Use Fig. 7 to determine the resistance of the component when the potential difference is 0.80 V. (c) State the approximate resistance of the component when the potential difference is -2.0 V.

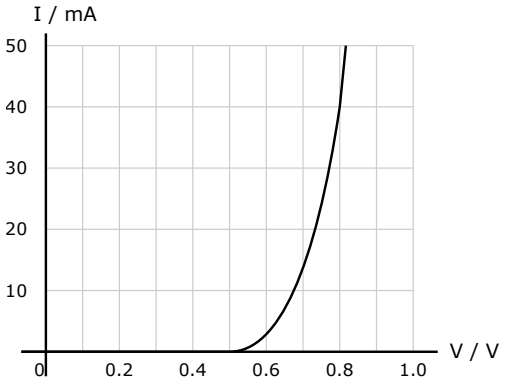

Fig. 7.

8. A negative temperature coefficient (NTC) thermistor is used in a temperature sensor. (a) State how the resistance of the thermistor changes as its temperature increases. (b) At 20 °C, the thermistor has a resistance of 2500 Ω. It is connected across a 10 V power supply. Calculate the power dissipated by the thermistor. (c) At 80 °C, the current in the thermistor is 20 mA when connected to the same 10 V supply. Calculate the resistance at 80 °C.

9. A light-dependent resistor (LDR) is used in a circuit to monitor the brightness of a laser beam. (a) State the effect on the resistance of the LDR when the light intensity of the laser beam decreases. (b) When the laser is switched off, the LDR is connected to a 5.0 V direct current (d.c.) supply and the current is 2.5 μA. Calculate the resistance of the LDR in the dark. (c) When the laser beam illuminates the LDR, the power dissipated by the LDR increases to 0.15 W for the same 5.0 V potential difference. Calculate the new current in the LDR. (d) Calculate the total number of electrons passing through the LDR in a time of 45 s when it is illuminated by the laser beam.

10. A uniform cylindrical wire X has length L, diameter d and resistance R. Wire Y is made of the same material. It has length $4L$ and diameter $d/2$. (a) Express the cross-sectional area of wire Y in terms of the cross-sectional area of wire X. (b) Determine the resistance of wire Y in terms of R. (c) If $R = 2.5$ Ω, calculate the resistance of wire Y.

11. (a) Define electric current. (b) An alpha particle beam consists of alpha particles, each with a charge of $+2e$. The beam carries a current of 1.5 nA. Calculate the number of alpha particles passing a fixed point in the beam in a time of 1.0 minute.

12. A metal wire has a square cross-section of side 0.40 mm. The current in the wire is 2.1 A and the average drift speed of the charge carriers is 0.15 mm s^{-1}. (a) Calculate the cross-sectional area of the wire in m^2. (b) Calculate the number density of the charge carriers in the metal.

13. An electric heater is constructed using a wire of resistivity 1.3×10^{-6} Ω m. The heater operates at a potential difference of 230 V and dissipates a power of 2.5 kW. (a) Calculate the resistance of the heater. (b) The wire has a diameter of 0.55 mm. Calculate the length of the wire used in the heater.

14. A direct current (d.c.) electric motor in a machine is connected to a power supply by a uniform copper cable. The cable has a length of 1.2 m and a cross-sectional area of 3.0×10^{-5} m^2. The current in the cable is 120 A.
    The resistivity of copper is 1.7×10^{-8} Ω m and the number density of free electrons in copper is 8.5×10^{28} m^{-3}.
    (a) Calculate the total charge passing a given point in the cable in a time of 4.0 s. (b) Calculate the resistance of the cable. (c) Calculate the potential difference across the ends of the cable. (d) Calculate the electrical power dissipated as thermal energy in the cable. (e) Determine the average drift speed of the free electrons in the cable.

15. (a) The charge on a charge carrier is quantised. Explain what is meant by this statement. (b) A student measures the net charge on three small oil drops, A, B and C. The measured charges are: Drop A: 3.2×10^{-19} C Drop B: 4.0×10^{-19} C Drop C: 6.4×10^{-19} C State and explain which of these drops has a measured charge that is not physically possible.

16. A portable water heater is rated at 12 V, 150 W. (a) Calculate the current in the heater when it operates at its rated voltage. (b) The heater is switched on for 3.5 minutes. Calculate the total charge that flows through the heater. (c) Calculate the number of electrons that pass through the heater in this time. (d) Define the potential difference across the heater.

17. A copper wire and an aluminium wire have the same length and the same resistance. The resistivity of copper is 1.7×10^{-8} Ω m and the resistivity of aluminium is 2.7×10^{-8} Ω m. (a) Calculate the ratio $\frac{\text{cross-sectional area of aluminium wire}}{\text{cross-sectional area of copper wire}}$. (b) Calculate the ratio $\frac{\text{diameter of aluminium wire}}{\text{diameter of copper wire}}$.

18. A semiconductor diode operates in a circuit. (a) When the diode is forward-biased, the potential difference across it is 0.75 V and the current is 25 mA. Calculate the power dissipated by the diode. (b) The diode is then reverse-biased with a potential difference of 5.0 V. The resistance of the diode is now 1.0×10^{8} Ω. Calculate the current in the diode.

19. A filament lamp has a non-linear $I - V$ characteristic. (a) When the potential difference is 3.0 V, the current is 1.5 A. Calculate the resistance. (b) When the potential difference is 12.0 V, the current is 4.0 A. Calculate the resistance. (c) By considering the temperature of the filament, explain the difference between the two resistance values.

20. A beam of electrons in a vacuum tube constitutes an electric current of 8.5 mA. (a) State what is meant by the quantisation of charge. (b) Calculate the time required for a total charge of 4.0 C to be carried by the electron beam. (c) The beam has a circular cross-section of diameter 2.0 mm. The electrons travel with an average velocity of 1.5×10^{7} m s^{-1}. Calculate the number density of the electrons in the beam.

21. A metal track on a printed circuit board has a rectangular cross-section of width 1.5 mm and thickness 40 μm. The length of the track is 8.0 cm. The metal has a resistivity of 1.8×10^{-8} Ω m. (a) Calculate the cross-sectional area of the track in m^2. (b) Calculate the resistance of the track.

22. A fast charger delivers a constant direct current of 32 A to an electric vehicle battery for a time of 45 minutes. The potential difference across the battery is 400 V.
(a) Calculate the total charge transferred to the battery. (b) Calculate the number of electrons equivalent to this total charge. (c) Calculate the total electrical energy transferred to the battery. (d) The charger is connected to the battery by a cable containing two identical copper wires. One wire carries the current to the battery and the other carries the return current. The length of the cable is 3.5 m. The total power dissipated as thermal energy in the two wires must not exceed 65 W when carrying the 32 A current. (i) Calculate the maximum allowable total resistance of the two wires. (ii) The resistivity of copper is 1.7×10^{-8} Ω m. Calculate the minimum required cross-sectional area of each copper wire.

23. A wire of diameter D carries a current I. The average drift speed of the free electrons is v. The wire is joined end-to-end to another wire of the same material but with diameter $3D$. The current I is the same in both wires. (a) State the relationship between the cross-sectional area of a wire and its diameter. (b) Determine the average drift speed of the electrons in the second wire in terms of v.

24. The number density of free electrons in silver is 5.8×10^{28} m^{-3}. A silver wire of length 0.50 m has a current of 2.5 A. The average drift speed of the electrons is 3.5×10^{-5} m s^{-1}. (a) Calculate the cross-sectional area of the wire. (b) Calculate the diameter of the wire.

25. (a) State the relationship between charge Q, current I and time t. (b) A metallic conductor has a constant resistance of 8.5 Ω. It is connected to a 17 V direct current (d.c.) supply. (i) Calculate the current in the conductor. (ii) Calculate the total charge that flows through the conductor in 3.0 minutes. (iii) Calculate the number of electrons that correspond to this charge.

Advanced Practice（培优）· 20

1. A composite wire consists of two cylindrical sections, X and Y, joined end-to-end. Both sections are made of the same metal. Section X has diameter d and section Y has diameter $3d$. A constant direct current I exists in the wire. (a) Explain why the current in section X is equal to the current in section Y. (b) Show that the ratio of the average drift speed of the free electrons in section X to that in section Y is 9.

2. An ion beam consists of doubly ionised positive ions (charge $+2e$). The beam carries a current of 3.2×10^{-6} A. (a) Calculate the number of ions passing a fixed point in the beam in a time of 5.0 minutes. (b) The potential difference accelerating the ions is 4.5×10^{3} V. Calculate the total energy transferred to the ions in this time.

3. The current-voltage ($I-V$) characteristic of a filament lamp is non-linear. At a potential difference of 2.0 V, the current is 0.40 A. At 6.0 V, the current is 0.80 A. (a) Calculate the change in the resistance of the lamp when the potential difference increases from 2.0 V to 6.0 V. (b) Explain, in terms of the charge carriers and lattice, why this change in resistance occurs. (c) Calculate the ratio of the power dissipated at 6.0 V to the power dissipated at 2.0 V.

4. A cylindrical metal wire has initial length L and resistance R. The wire is drawn through a die so that its length increases to $2.5L$. The volume and resistivity of the metal remain constant. (a) Show that the new resistance of the wire is $6.25R$. (b) The original wire had a diameter of 0.80 mm and a length of 1.5 m. Its resistance was 0.12 Ω. Calculate the resistivity of the metal.

5. A negative temperature coefficient (NTC) thermistor is connected to a constant voltage supply. (a) Explain why the resistance of the thermistor decreases as its temperature increases, referring to the equation $I = Anvq$. (b) The thermistor is at 20°C and carries a current of 15 mA. At 50°C, the current is 45 mA. The potential difference remains constant. Determine the ratio of its resistance at 50°C to its resistance at 20°C.

6. A block of conducting material has dimensions 2.0 cm × 3.0 cm × 15.0 cm. The resistivity of the material is 8.0×10^{-4} Ω m. (a) Calculate the resistance of the block when a potential difference is applied across the two 2.0 cm × 3.0 cm faces. (b) Calculate the resistance when the potential difference is applied across the two 3.0 cm × 15.0 cm faces.

7. A heating element is made of a uniform wire of resistance 48 Ω. It is connected to a 240 V supply. (a) Calculate the power dissipated by the heating element. (b) The wire is cut into two equal halves. The two halves are then twisted together to form a single wire of half the original length and twice the cross-sectional area. This new wire is connected to the same 240 V supply. Determine the new power dissipated.

8. A light-dependent resistor (LDR) is connected across a 9.0 V constant voltage source. In a dark room, the power dissipated by the LDR is 1.5×10^{-3} W. When a light is turned on, the current in the LDR becomes 12 mA. (a) Calculate the resistance of the LDR in the dark room. (b) Calculate the resistance of the LDR when the light is on. (c) State how the number density of charge carriers in the LDR changes when the light is turned on.

9. A silver wire has a cross-sectional area of 2.5×10^{-6} m^2 and carries a current of 5.4 A. The number density of free electrons in silver is 5.9×10^{28} m^{-3}. (a) Calculate the average drift speed of the free electrons in the wire. (b) The current is maintained at 5.4 A for 2.0 hours. Calculate the total number of free electrons that pass a given point in the wire during this time.

10. The variation with potential difference V of the current I for a semiconductor diode is shown in Fig. 10. (a) Use Fig. 10 to determine the resistance of the diode at $V = 0.80$ V. (b) State and explain how the resistance of the diode changes as V increases from 0.60 V to 1.0 V.

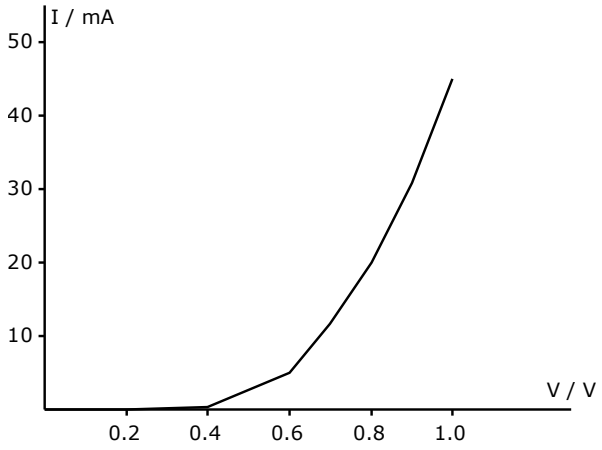

Fig. 10.

11. The potential difference across a component is 12 V. (a) Define potential difference. (b) Calculate the energy transferred when 4.5×10^{19} electrons pass through the component.

12. A student states that Ohm's law is given by the equation $V = IR$. (a) State Ohm's law correctly. (b) A filament lamp has a resistance of 15 Ω when the current is 0.20 A. Calculate the potential difference across the lamp. (c) Explain why the lamp does not obey Ohm's law.

13. A hollow cylindrical copper tube has an inner radius of 4.0 mm, an outer radius of 6.0 mm, and a length of 1.2 m. The resistivity of copper is 1.7×10^{-8} Ω m. (a) Calculate the cross-sectional area of the copper in the tube. (b) Calculate the electrical resistance of the tube from one end to the other.

14. A power cable has a resistance of 0.45 Ω per kilometre. The cable transmits a current of 250 A over a distance of 16 km. (a) Calculate the total resistance of the cable. (b) Calculate the power dissipated as heat in the cable. (c) The current is reduced to 125 A while transmitting the same total power. Determine the new power dissipated in the cable and state the factor by which the power loss is reduced.

15. Fig. 15 shows the $I - V$ characteristic for a uniform metal wire of length 2.5 m and diameter 0.30 mm at a constant temperature. (a) Determine the resistance of the wire. (b) Calculate the resistivity of the metal.

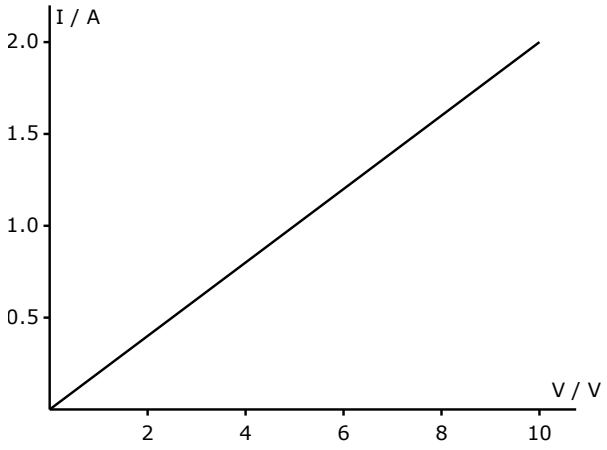

Fig. 15.

16. Two wires, P and Q, have the same dimensions and carry the same current. Wire P is made of a metal with a charge carrier number density of n_P. Wire Q is made of a semiconductor with a charge carrier number density of n_Q. The ratio $n_P/n_Q = 1.5 \times 10^5$. (a) State the ratio of the average drift speed of the charge carriers in wire P to that in wire Q. (b) The current in wire P is 1.2 A. Calculate the total charge passing a point in wire P in 3.0 hours.

17. A uniform rectangular track of conducting material has length L, cross-sectional area A and resistivity ho. A constant potential difference V is applied across the length of the track. The material has a free electron number density n and the elementary charge is e.
(a) By using the equations for electric current and resistance, show that the average drift speed v of the free electrons in the track is given by $v = \frac{V}{\rho Lne}$.
(b) A particular track made of this material has length 2.5×10^{-2} m, resistivity 4.8×10^{-5} Ω m and $n = 6.0 \times 10^{25}$ m^{-3}. The potential difference applied across its length is 3.6 V. Calculate the average drift speed of the free electrons.
(c) The power dissipated by this track is 0.54 W. Calculate the cross-sectional area of the track.
(d) A second track made of the same material has the same length L but a different cross-sectional area. It dissipates 1.08 W of power when connected to the same 3.6 V potential difference. State the average drift speed of the free electrons in this second track.

18. A filament lamp is rated as '12 V, 36 W'. When operating at its rated voltage, it has reached its normal working temperature. (a) Calculate the resistance of the lamp at its normal working temperature. (b) When the lamp is first switched on, the initial current is 18 A. Calculate the initial resistance of the cold filament. (c) Suggest why the current drops rapidly from 18 A to its normal operating value.

19. An electron beam in a vacuum tube carries a current of 4.8 μA. The beam has a circular cross-section of diameter 1.2 mm. The electrons travel at a constant speed of 2.4×10^7 m s^{-1}. (a) Calculate the number density of electrons in the beam. (b) Calculate the time taken for 1.0 C of charge to be transported by the beam.

20. A student measures the resistance R of different lengths L of a nichrome wire. The results are plotted in Fig. 20. The wire has a uniform diameter of 0.46 mm. (a) Determine the gradient of the line in Fig. 20. (b) Use your answer in (a) to calculate the resistivity of nichrome.

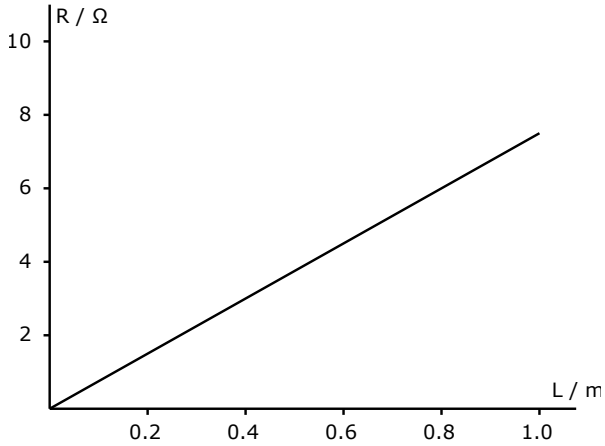

Fig. 20.

10 D.C. circuits（直流电路）

Principles of direct current circuits, including electromotive force, internal resistance, Kirchhoff's laws, and potential dividers.

Rules 规则

- Electromotive force (e.m.f.) is the energy transferred from other forms to electrical per unit charge driving charge around a complete circuit (电动势).
- Potential difference (p.d.) is the energy transferred from electrical to other forms per unit charge between two points (电势差).
- The terminal p.d. V of a source with e.m.f. E and internal resistance r delivering a current I is given by $V = E - Ir$ (路端电压与内阻).
- Kirchhoff's first law states that the sum of currents entering a junction equals the sum of currents leaving it, which is a consequence of the conservation of charge (基尔霍夫第一定律与电荷守恒).
- Kirchhoff's second law states that in any closed loop, the sum of e.m.f.s equals the sum of p.d.s, which is a consequence of the conservation of energy (基尔霍夫第二定律与能量守恒).
- For resistors in series, the combined resistance is $R = R_1 + R_2 + ...$; for resistors in parallel, the combined resistance is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ...$ (串联与并联电阻).
- In a potential divider, the output voltage V_{out} across a resistor R_1 in series with R_2 from a supply V_{in} is $V_{\text{out}} = V_{\text{in}} \times \frac{R_1}{R_1+R_2}$ (分压器).

Worked Examples 例题

Example 10.1

A battery of e.m.f. 9.0 V and internal resistance r is connected to a 4.0 Ω resistor. The current in the circuit is 2.0 A. Calculate the internal resistance r.

Solution

Using the equation $E = I(R + r)$, substitute the given values: $9.0 = 2.0 \times (4.0 + r)$. Expanding this gives $9.0 = 8.0 + 2.0r$. Solving for r yields $2.0r = 1.0$, so $r = 0.50$ Ω.

Example 10.2

Two cells, one of e.m.f. 6.0 V and internal resistance 1.0 Ω, and another of e.m.f. 2.0 V and internal resistance 0.50 Ω, are connected in series opposing each other with a 4.5 Ω resistor. Calculate the current in the circuit.

Solution

According to Kirchhoff's second law, $\Sigma E = \Sigma IR$. The net e.m.f. in the loop is $6.0 - 2.0 = 4.0$ V. The total resistance in the circuit is $1.0 + 0.50 + 4.5 = 6.0$ Ω. Therefore, $4.0 = I \times 6.0$, which gives $I = \frac{4.0}{6.0} = 0.67$ A.

Example 10.3

A potential divider consists of a light-dependent resistor (LDR) and a fixed resistor of 5.0 kΩ connected in series across a 12 V supply of negligible internal resistance. In bright light, the LDR has a resistance of 1.0 kΩ. Calculate the potential difference across the fixed resistor in bright light.

Solution

Use the potential divider equation $V_{\text{out}} = V_{\text{in}} \times \frac{R_{\text{fixed}}}{R_{\text{fixed}} + R_{\text{LDR}}}$. Substituting the values gives $V_{\text{out}} = 12 \times \frac{5.0}{5.0+1.0} = 12 \times \frac{5.0}{6.0} = 10$ V.

Example 10.4

A circuit contains a 12 V power supply with negligible internal resistance connected to a 2.0 Ω resistor in series with a parallel combination of a 3.0 Ω and a 6.0 Ω resistor. Calculate the total current drawn from the supply.

Solution

First, find the equivalent resistance R_p of the parallel combination using $\frac{1}{R_p} = \frac{1}{3.0} + \frac{1}{6.0} = \frac{3.0}{6.0}$. This gives $R_p = 2.0$ Ω. The total resistance of the circuit is $R_{\text{total}} = 2.0 + 2.0 = 4.0$ Ω. The total current is $I = \frac{V}{R_{\text{total}}} = \frac{12}{4.0} = 3.0$ A.

考点总览 Key Points

- □ 我能识别并规范绘制常用电路符号 (circuit symbols)，并能分析与解读电路图。
- □ 我理解电动势 (e.m.f.) 和电势差 (p.d.) 的能量转化本质与定义公式（$E = \frac{W}{Q}$）。
- □ 我能分析电源内阻 (internal resistance) 对路端电压 (terminal p.d.) 的影响（$V = E - Ir$）。
- □ 我掌握基尔霍夫第一定律 (Kirchhoff's first law) 及其电荷守恒 (conservation of charge) 本质（$\sum I_{in} = \sum I_{out}$）。
- □ 我掌握基尔霍夫第二定律 (Kirchhoff's second law) 及其能量守恒 (conservation of energy) 本质（$\sum E = \sum IR$）。
- □ 我会运用基尔霍夫定律推导并计算串联和并联电阻总阻值（如 $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$）。
- □ 我能综合运用基尔霍夫定律列方程求解简单电路中的电流和电压未知量。
- □ 我理解分压电路 (potential divider) 的工作原理并能计算输出电压（$V_{out} = \frac{R_2}{R_1+R_2}V_{in}$）。
- □ 我能解释热敏电阻 (thermistor) 和光敏电阻 (LDR) 在分压电路中对温度和光强的传感控制。
- □ 我熟悉电位计 (potentiometer) 比较电势差的原理及检流计 (galvanometer) 在零差法 (null method) 中的应用。

Basic Drill（入门综合）· 50

1. Define electromotive force (e.m.f.) of an electrical source.

2. Distinguish between electromotive force (e.m.f.) and potential difference (p.d.) in terms of energy transformations.

3. A battery of e.m.f. 9.0 V delivers a steady current of 0.50 A for a time of 120 s. Calculate the total energy transferred by the battery to the charge passing through it.

4. A cell of e.m.f. 6.0 V and internal resistance 0.80 Ω is connected across an external resistor of resistance 4.2 Ω. Calculate the terminal potential difference across the cell.

5. A battery has an e.m.f. of 12.0 V. When a current of 2.5 A is drawn from the battery, its terminal potential difference drops to 10.5 V. Calculate the internal resistance of the battery.

6. State Kirchhoff's first law and name the fundamental physical quantity that is conserved according to this law.

7. Four wires meet at a single junction in a circuit. Currents $I_1 = 3.2$ A and $I_2 = 1.8$ A enter the junction. Current $I_3 = 2.6$ A leaves the junction. Calculate the magnitude and state the direction of the current I_4 in the fourth wire.

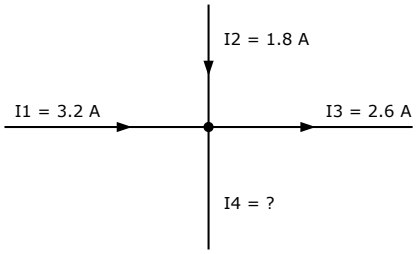

Fig. 7. Currents meeting at a junction.

8. State Kirchhoff's second law and name the fundamental physical quantity that is conserved according to this law.

9. Three resistors with resistances of 15 Ω, 30 Ω, and 45 Ω are connected in series. Calculate the total combined resistance of the network.

10. Three resistors of resistances 20 Ω, 30 Ω, and 60 Ω are connected in parallel. Calculate the equivalent resistance of this parallel combination.

11. Two identical resistors, each of resistance R, are connected in parallel. This combination is connected in series with a third identical resistor of resistance R. The total equivalent resistance of the entire arrangement is 36 Ω. Calculate the resistance R.

12. Two resistors of resistances 4.0 Ω and 6.0 Ω are connected in parallel across a power supply of e.m.f. 12 V and negligible internal resistance. Calculate the total current supplied by the power supply.

13. Two cells of e.m.f. $E_1 = 6.0$ V and $E_2 = 1.5$ V, each having negligible internal resistance, are connected in series with their polarities opposing each other across a resistor of resistance 15 Ω. Calculate the current in the resistor.

14. Explain why the terminal potential difference across a cell is less than its electromotive force when a non-zero current is drawn from the cell.

15. A potential divider consists of two resistors of resistances $R_1 = 2.0\ \text{k}\Omega$ and $R_2 = 8.0\ \text{k}\Omega$ connected in series across a 10 V power supply of negligible internal resistance. Calculate the output potential difference V_{out} across resistor R_2.

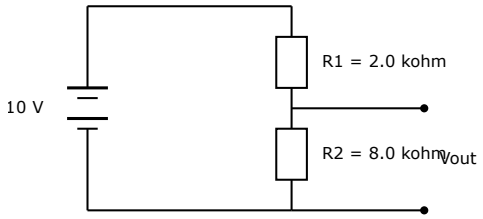

Fig. 15. Potential divider circuit with two resistors.

16. A potential divider circuit consists of a fixed resistor of resistance 120 Ω connected in series with a variable resistor across a 9.0 V supply of negligible internal resistance. Calculate the resistance of the variable resistor when the potential difference across the fixed resistor is 3.0 V.

17. A negative temperature coefficient (NTC) thermistor is connected in series with a fixed resistor across a d.c. supply of constant voltage. State and explain the change in the potential difference across the fixed resistor as the temperature of the thermistor increases.

18. A light-dependent resistor (LDR) is connected in series with a 5.0 kΩ fixed resistor across a 12 V d.c. supply of negligible internal resistance. When illuminated by bright light, the resistance of the LDR decreases to 1.0 kΩ. Calculate the output potential difference across the LDR under this bright light.

19. State what is meant by a null method in potentiometer measurements.

20. A potentiometer consists of a uniform resistance wire of length 1.00 m. A cell of known e.m.f. 1.50 V is balanced against a length of 60.0 cm of the wire. A second cell of unknown e.m.f. E_x gives a balance length of 84.0 cm. Calculate the e.m.f. E_x.

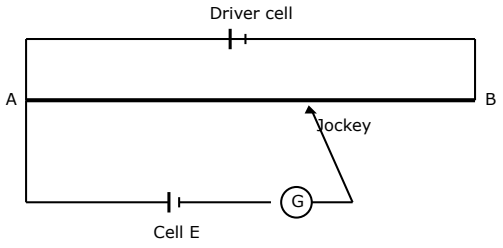

Fig. 20. Potentiometer circuit for comparing e.m.f.s.

21. Explain why measuring the e.m.f. of a cell using a potentiometer at the balance point is more accurate than using a standard voltmeter.

22. A cell of e.m.f. 1.5 V produces a current of 5.0 A when its terminals are connected directly together by a wire of negligible resistance. Calculate the internal resistance of the cell.

23. A battery of e.m.f. E and internal resistance r is connected to an external load resistor of resistance R. Write down an expression for the current I in the circuit in terms of E, R, and r.

24. Four identical resistors, each of resistance $R = 10\ \Omega$, are connected to form a closed square $ABCD$. Calculate the equivalent resistance measured between two diagonally opposite corners A and C.

25. A battery of e.m.f. 15 V and internal resistance 1.0 Ω is connected in a single loop to two external resistors of resistances 4.0 Ω and 5.0 Ω. Calculate the lost volts within the battery.

26. A battery of electromotive force (e.m.f.) 9.0 V and internal resistance 1.5 Ω is connected directly across a digital voltmeter of infinite internal resistance. State the reading shown on the voltmeter.

27. Three identical resistors, each of resistance 30 Ω, are connected together in parallel. Calculate the total combined resistance of this arrangement.

28. A potential divider consists of a 400 Ω resistor and a 600 Ω resistor connected in series across a 12 V power supply of negligible internal resistance. Calculate the potential difference across the 600 Ω resistor.

29. State the fundamental law of conservation on which Kirchhoff's second law is based.

30. A battery has an electromotive force (e.m.f.) of 6.0 V. Calculate the total energy transferred by the battery when a charge of 45 C is driven around a complete circuit.

31. A light-dependent resistor (LDR) and a fixed resistor of resistance 5.0 kΩ are connected in series across a 10 V d.c. supply of negligible internal resistance. In bright light, the resistance of the LDR is 1.2 kΩ. Calculate the potential difference across the fixed resistor in bright light.

32. Four wires meet at a single circuit junction. Currents of 2.5 A, 1.8 A, and 3.2 A flow towards the junction. A fourth wire carries the current away from the junction. Calculate the magnitude of the current in this fourth wire.

33. A cell of e.m.f. 1.50 V delivers a current of 0.40 A to an external circuit. The terminal potential difference across the cell is measured to be 1.34 V. Calculate the internal resistance of the cell.

34. A uniform resistance wire XY of length 1.00 m is connected in a potentiometer circuit to a driver cell of e.m.f. 2.00 V and negligible internal resistance. A test cell of e.m.f. 1.44 V produces a null deflection on the galvanometer at a balance length L along the wire from X. Calculate L.

35. Three resistors of resistances 12 Ω, 18 Ω, and 30 Ω are connected in series with one another. Calculate their total combined resistance.

36. Distinguish between electromotive force (e.m.f.) and potential difference (p.d.) in terms of energy transformations in a circuit.

37. A negative temperature coefficient (NTC) thermistor is connected in series with a fixed resistor across a constant voltage power supply. State and explain the effect on the potential difference across the fixed resistor when the temperature of the thermistor is increased.

38. Two resistors of resistances 6.0 Ω and 12 Ω are connected in parallel. This parallel pair is then connected in series with a 4.0 Ω resistor. Calculate the total resistance of this network.

39. A source of e.m.f. 12 V has an internal resistance of 0.80 Ω. When connected to an external load, a current of 2.5 A flows. Calculate the lost volts across the internal resistance.

40. In the context of a potentiometer circuit, state what is meant by a null method.

41. State Kirchhoff's first law.

42. A cell of e.m.f. 4.5 V and internal resistance 0.50 Ω is connected across a resistor of resistance 8.5 Ω. Calculate the current flowing in the circuit.

43. A potentiometer consists of a uniform wire of length 100 cm connected across a 3.00 V supply of negligible internal resistance. A balance point is obtained for an unknown cell at a length of 48.0 cm. Calculate the e.m.f. of the unknown cell.

44. Two resistors of resistances 20 Ω and 30 Ω are connected in parallel across a 12 V d.c. source of negligible internal resistance. Calculate the current passing through the 20 Ω resistor.

45. A potential divider consists of a fixed resistor $R_1 = 150\ \Omega$ and a variable resistor R_2 connected in series across a 9.0 V supply of negligible internal resistance. Calculate the resistance of R_2 required to give an output potential difference of 3.0 V across R_2.

46. A student plots a graph of terminal potential difference V on the vertical axis against current I on the horizontal axis for a cell. State what physical quantity is represented by the intercept on the vertical axis.

47. Four identical resistors, each of resistance 8.0 Ω, are arranged as two parallel pairs connected in series with each other. Calculate the total combined resistance of the circuit.

48. A light-dependent resistor (LDR) is connected in series with a fixed resistor across a constant potential difference. The output potential difference is measured across the LDR. State what happens to the output potential difference when the light intensity decreases.

49. A battery of e.m.f. E and internal resistance r is connected to an external load resistor of resistance R. Express the terminal potential difference V across the battery in terms of E, R, and r.

50. A single closed loop contains two batteries connected in opposition, with their positive terminals connected together, and a resistor of resistance 4.0 Ω. Battery 1 has an e.m.f. of 12 V and internal resistance 0.50 Ω. Battery 2 has an e.m.f. of 4.0 V and internal resistance 1.5 Ω. Calculate the magnitude of the current in the circuit.

Practice（综合练习）· 25

1. (a) Define the electromotive force (e.m.f.) of a source. (b) Distinguish between e.m.f. and potential difference (p.d.) in terms of energy considerations.

2. A battery of e.m.f. 9.0 V and internal resistance r is connected to a resistor of resistance 12 Ω. The current in the circuit is 0.70 A. Calculate: (a) the terminal potential difference across the battery, (b) the internal resistance r.

3. (a) State Kirchhoff's first law. (b) Explain how this law is a consequence of a conservation law. (c) At a junction in a circuit, three wires carry currents of 2.5 A, 1.8 A and I. The 2.5 A and 1.8 A currents flow towards the junction. Determine the magnitude and direction of current I.

4. (a) State Kirchhoff's second law. (b) State the physical quantity that is conserved in Kirchhoff's second law. (c) A closed loop in a circuit contains a cell of e.m.f. 12 V and two resistors of resistances R_1 and R_2 in series. The potential difference across R_1 is 4.5 V. The internal resistance of the cell is negligible. Calculate the potential difference across R_2.

5. Use Kirchhoff's laws to derive the formula for the combined resistance R of three resistors of resistances R_1, R_2 and R_3 connected in series.

6. Use Kirchhoff's laws to derive the formula for the combined resistance R of three resistors of resistances R_1, R_2 and R_3 connected in parallel.

7. Three resistors are connected in a circuit. Resistor A has a resistance of 15 Ω. Resistors B and C, each of resistance 10 Ω, are connected in parallel with each other. This parallel combination is connected in series with resistor A. Calculate the total combined resistance of the three resistors.

8. A battery of e.m.f. E and internal resistance r is connected to a variable resistor. When the resistance of the variable resistor is 8.0 Ω, the current in the circuit is 0.50 A. When the resistance is 3.0 Ω, the current is 1.0 A. Calculate: (a) the internal resistance r, (b) the e.m.f. E.

9. A potential divider circuit consists of a power supply of e.m.f. 9.0 V and negligible internal resistance, connected in series with two resistors of resistances 47 Ω and 33 Ω. Calculate the potential difference across the 33 Ω resistor.

10. A potential divider consists of a fixed resistor and a light-dependent resistor (LDR) connected in series across a cell of constant e.m.f. and negligible internal resistance. A voltmeter is connected in parallel across the fixed resistor. Explain how the reading on the voltmeter changes when the light intensity incident on the LDR is increased.

11. A potential divider circuit contains a thermistor and a fixed resistor of resistance 2.0 kΩ connected in series across a 6.0 V supply of negligible internal resistance. At 20 °C, the resistance of the thermistor is 5.0 kΩ. At 100 °C, its resistance is 0.50 kΩ. Calculate the change in the potential difference across the fixed resistor when the temperature of the thermistor increases from 20 °C to 100 °C.

12. (a) State the principle of a potentiometer used to compare potential differences. (b) Explain the use of a galvanometer in a null method for a potentiometer circuit.

13. A potentiometer is set up using a uniform wire of length 1.00 m. A standard cell of e.m.f. 1.50 V gives a balance length of 0.60 m. A test cell gives a balance length of 0.76 m. Calculate the e.m.f. of the test cell.

14. A circuit consists of a single loop containing a resistor of resistance 3.2 Ω and two cells, A and B. Cell A has an e.m.f. of 6.0 V and internal resistance 0.50 Ω. Cell B has an e.m.f. of 2.0 V and internal resistance 0.30 Ω. The cells are connected such that their electromotive forces oppose each other. Calculate: (a) the current in the circuit, (b) the terminal potential difference of cell A.

15. The circuit shown in Fig. 15 contains a battery of e.m.f. 12 V and negligible internal resistance, connected to three resistors R_1, R_2 and R_3. The resistances are $R_1 = 4.0\ \Omega$, $R_2 = 6.0\ \Omega$ and $R_3 = 3.0\ \Omega$. Calculate: (a) the combined resistance of the circuit, (b) the current in R_1, (c) the current in R_2.

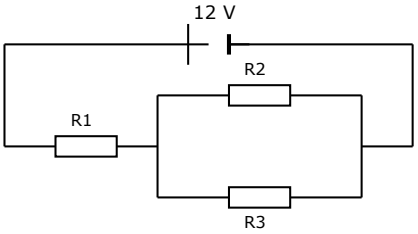

Fig. 15.

16. A student performs an experiment to determine the e.m.f. E and internal resistance r of a cell. The student measures the terminal potential difference V across the cell for different values of current I. The results are shown in Table 66.1.
Table 66.1 | I / A | V / V | |—|—| | 0.50 | 4.1 | | 2.00 | 3.2 |
(a) State the equation relating V, E, I and r. (b) Use the data to calculate r. (c) Determine E.

17. In the circuit shown in Fig. 17, a battery of e.m.f. 15 V and negligible internal resistance is connected to a potential divider consisting of two resistors of resistances 120 Ω and 80 Ω. A load resistor of resistance 240 Ω is connected in parallel with the 80 Ω resistor. Calculate: (a) the combined resistance of the 80 Ω and 240 Ω resistors, (b) the potential difference across the load resistor.

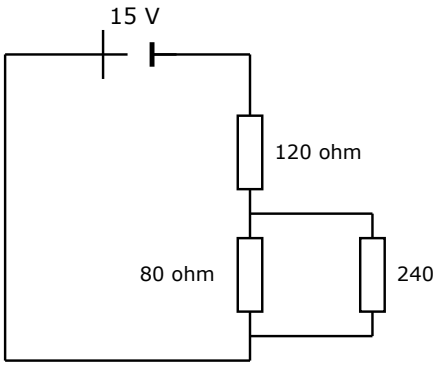

Fig. 17.

18. A potentiometer circuit is used to measure an unknown e.m.f. The circuit consists of a driver cell of e.m.f. 4.0 V and internal resistance 2.0 Ω, connected in series with a uniform resistance wire of length 1.00 m and resistance 14.0 Ω. Calculate: (a) the current in the resistance wire, (b) the potential difference across the entire length of the resistance wire, (c) the balance length for a test cell of e.m.f. 1.5 V.

19. Two identical cells, each of e.m.f. 1.5 V and internal resistance 0.40 Ω, are connected to a resistor of resistance R. (a) The two cells are first connected in series with the resistor R. The current in the circuit is 0.60 A. Calculate R. (b) The two cells are then connected in parallel with each other, and this combination is connected to the same resistor R. Calculate the new current in the circuit.

20. A student is designing a potential divider circuit to turn on a cooling fan when the temperature rises. The circuit uses a battery of constant e.m.f., a fixed resistor and a thermistor. The output potential difference is taken across the fixed resistor. (a) Explain why a negative temperature coefficient (NTC) thermistor is suitable for this purpose. (b) The e.m.f. of the battery is 9.0 V. At a certain temperature, the thermistor has a resistance of 1.5 kΩ and the output potential difference across the fixed resistor is 3.6 V. Calculate the resistance of the fixed resistor.

21. The circuit in Fig. 21 consists of two loops sharing a common middle branch. The left branch contains a cell of e.m.f. E_1 and a resistor of 4.0 Ω. The right branch contains a cell of e.m.f. 6.0 V and a resistor of 8.0 Ω. The middle branch contains a resistor of 5.0 Ω. The current in the middle branch is 2.0 A flowing downwards. The current in the right branch is 0.50 A flowing downwards. The positive terminals of both cells are connected to the top wire. All cells have negligible internal resistance. Calculate: (a) the magnitude and direction of the current in the left branch, (b) the e.m.f. E_1.

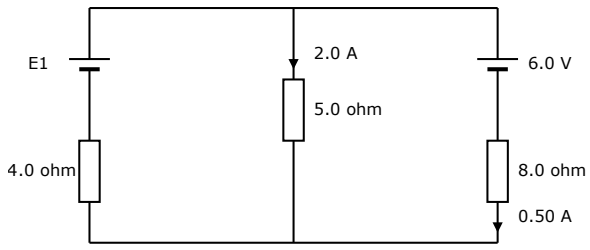

Fig. 21.

22. A potentiometer is used to determine the e.m.f. of a test cell. The balance length is found to be 55.0 cm. The driver cell in the primary circuit has a small internal resistance. State and explain the effect on the balance length if the driver cell is replaced by another cell with the same e.m.f. but a larger internal resistance.

23. Three identical resistors, each of resistance 24 Ω, are available. (a) Calculate the total combined resistance when all three resistors are connected in parallel. (b) Calculate the total combined resistance when two of the resistors are connected in parallel, and the third is connected in series with this combination.

24. A battery of e.m.f. 12.0 V and internal resistance 0.50 Ω is connected to a resistor. The terminal potential difference of the battery is measured to be 10.5 V. (a) Calculate the current in the circuit. (b) Calculate the resistance of the resistor connected to the battery.

25. A circuit is set up to measure a resistance R. It consists of a power supply of e.m.f. 6.0 V connected across two parallel branches. In the first branch, a 200 Ω resistor and a 300 Ω resistor are connected in series, with the 200 Ω resistor connected to the positive terminal. In the second branch, a 400 Ω resistor and an unknown resistor R are connected in series, with the 400 Ω resistor connected to the positive terminal. A sensitive voltmeter is connected between the junction of the resistors in the first branch and the junction of the resistors in the second branch. The voltmeter reads 0 V. Calculate the value of R.

Advanced Practice（培优）· 20

1. A battery of electromotive force (e.m.f.) 12.0 V and internal resistance r is connected in series with a 4.0 Ω resistor. In a time of 15 s, the energy transferred to the 4.0 Ω resistor is 240 J. (a) Calculate the current in the circuit. (b) Calculate the internal resistance r. (c) State the difference between e.m.f. and potential difference (p.d.) in terms of energy considerations.

2. A student investigates a cell of e.m.f. E and internal resistance r. A variable resistor is connected across the terminals of the cell. The student plots a graph of the terminal potential difference V against the current I. The line of best fit is straight and passes through the coordinates $(0\ \text{A}, 1.50\ \text{V})$ and $(2.5\ \text{A}, 0.75\ \text{V})$. (a) Determine E and r. (b) Calculate the maximum power that this cell can deliver to the external variable resistor.

3. Two batteries, A and B, are connected in a single series loop with a 5.0 Ω resistor. Battery A has an e.m.f. of 12.0 V and an internal resistance of 1.0 Ω. Battery B has an e.m.f. of 4.0 V and an internal resistance of 2.0 Ω. The positive terminal of battery A is connected to the positive terminal of battery B through the resistor, such that the two e.m.f.s oppose each other. (a) Calculate the current in the circuit. (b) Calculate the terminal potential difference across battery B.

4. Use Kirchhoff's laws to derive the formula for the combined resistance R of three resistors of resistances R_1, R_2, and R_3 connected in parallel.

5. The circuit in the figure contains two cells and a resistor R. Cell 1 has an e.m.f. of 10.0 V and internal resistance 1.0 Ω. Cell 2 has an e.m.f. of 4.0 V and internal resistance 2.0 Ω. The central resistor R has a resistance of 2.0 Ω. (a) Calculate the current passing through resistor R. (b) Calculate the power dissipated in the internal resistance of Cell 2.

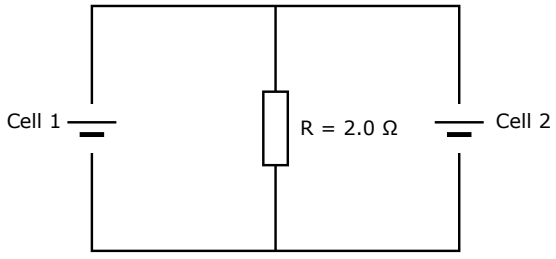

Fig. 5. A two-loop circuit with two cells and a shared resistor.

6. A potential divider consists of a 12.0 V power supply of negligible internal resistance connected in series with two fixed resistors of resistances 400 Ω and 600 Ω. (a) Calculate the potential difference across the 600 Ω resistor. (b) A voltmeter of finite resistance 1200 Ω is now connected across the 600 Ω resistor. Calculate the new reading on the voltmeter. (c) Calculate the percentage decrease in the potential difference across the 600 Ω resistor caused by connecting the voltmeter.

7. A potentiometer circuit is used to determine the e.m.f. E_X of a test cell. A driver cell of e.m.f. 4.0 V and negligible internal resistance is connected across a uniform resistance wire AB of length 1.00 m. The test cell of e.m.f. E_X and internal resistance 0.50 Ω is connected in series with a galvanometer. The null point is found at a distance of 0.65 m from A. (a) State the reading on the galvanometer at the null point. (b) Calculate E_X. (c) A 2.0 Ω resistor is now connected in parallel with the test cell. Explain why the balance length will now be less than 0.65 m.

8. A light-dependent resistor (LDR) and a fixed resistor of resistance R are connected in series across a 9.0 V battery of negligible internal resistance. The potential difference across the LDR is monitored. In dark conditions, the LDR has a resistance of 25 kΩ. In bright sunlight, its resistance is 400 Ω. (a) The p.d. across the LDR is required to be 6.0 V in dark conditions. Calculate R. (b) Calculate the p.d. across the LDR in bright sunlight using this value of R. (c) Explain why the p.d. across the fixed resistor increases as the light intensity increases.

9. A potential divider is designed to activate a cooling fan when the temperature exceeds 30°C. The circuit contains a 5.0 V power supply of negligible internal resistance, a negative temperature coefficient (NTC) thermistor, and a variable resistor. The fan is connected in parallel with the variable resistor and turns on when the p.d. across it reaches 2.5 V. At 30°C, the thermistor has a resistance of 1.8 kΩ. (a) Calculate the required resistance of the variable resistor to turn the fan on at exactly 30°C. (b) The temperature increases to 45°C. State and explain what happens to the p.d. across the variable resistor. (c) Suggest why a variable resistor is used instead of a fixed resistor in this circuit.

10. A driver cell of e.m.f. 6.0 V and internal resistance 1.2 Ω is connected across a uniform resistance wire AB of length 1.50 m and resistance 4.8 Ω. A test cell of e.m.f. E is connected to a galvanometer. The null point is found at a distance of 0.85 m from A. (a) Calculate the current in wire AB. (b) Calculate the potential difference across wire AB. (c) Determine E.

11. The figure shows a bridge circuit. A 12 V supply of negligible internal resistance is connected across nodes A and B. An ammeter of negligible resistance is connected between nodes C and D. (a) Calculate the equivalent resistance of the entire circuit. (b) Calculate the current passing through the 2.0 Ω resistor connected between A and C. (c) Determine the magnitude and direction of the current passing through the ammeter.

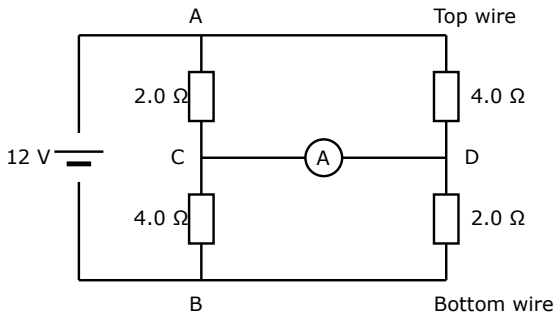

Fig. 11. A bridge circuit with an ammeter connected across the middle.

12. A battery of e.m.f. 9.0 V and internal resistance r is connected to a variable resistor of resistance R. (a) State Kirchhoff's second law. (b) When $R = 2.0\ \Omega$, the current in the circuit is 2.4 A. Calculate r. (c) Calculate the power dissipated in the variable resistor when $R = 2.0\ \Omega$. (d) The value of R is changed to 1.75 Ω. Calculate the new power dissipated in R and state how it compares to your answer in (c).

13. A circuit contains a 24 V supply of negligible internal resistance connected to three resistors. A resistor $R_1 = 8.0\ \Omega$ is in series with a parallel combination of $R_2 = 12\ \Omega$ and $R_3 = 24\ \Omega$. (a) Calculate the total resistance of the circuit. (b) Calculate the current from the supply. (c) Calculate the energy dissipated in R_2 in 5.0 minutes.

14. A potentiometer is used to compare the e.m.f. of two cells, E_1 and E_2. The balance length for E_1 is 45.0 cm. When E_1 is replaced by E_2, the balance length is 60.0 cm. (a) State the principle of the potentiometer. (b) Calculate the ratio E_1/E_2. (c) The driver cell has an e.m.f. of 5.0 V and the uniform resistance wire is 100 cm long. If $E_2 = 2.4$ V, calculate the terminal p.d. across the entire potentiometer wire. (d) Suggest why the terminal p.d. of the driver cell is less than its e.m.f. of 5.0 V.

15. Use Kirchhoff's laws to derive the formula for the combined resistance R of three resistors of resistances R_1, R_2, and R_3 connected in series.

16. The figure shows a circuit with five identical 10 Ω resistors arranged in a square with one diagonal. A 5.0 V cell of negligible internal resistance is connected across terminals A and B. (a) Calculate the total equivalent resistance of the circuit. (b) Calculate the total current supplied by the cell. (c) Calculate the current in the resistor connected between nodes D and C.

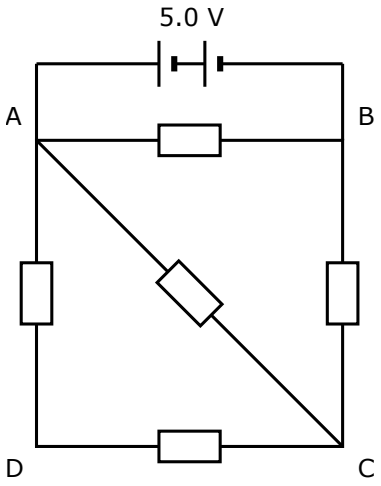

Fig. 16. Five identical resistors connected between nodes A, B, C, and D.

17. A rotary potentiometer is used as a potential divider. The resistance track has a total resistance of 5.0 kΩ and is connected across a 12 V supply of negligible internal resistance. The wiper is connected to an output terminal. A load resistor of 10 kΩ is connected between the wiper and the 0 V terminal. The wiper is set to the exact midpoint of the resistance track. (a) Calculate the effective resistance of the lower part of the potential divider, including the load resistor. (b) Calculate the output potential difference across the load resistor. (c) The load resistor is removed. State the new output potential difference.

18. Four identical cells, each of e.m.f. 1.5 V and internal resistance 0.20 Ω, are connected in series to form a battery. The battery is connected to a lamp of resistance 5.2 Ω. (a) Calculate the current in the lamp. (b) One of the cells is accidentally connected in reverse. Calculate the new current in the lamp. (c) Calculate the terminal potential difference of the reversed cell in this new circuit.

19. In a complex circuit, five wires meet at a single junction. The currents in three of the wires are $I_1 = 2.0$ A entering the junction, $I_2 = 4.5$ A leaving the junction, and $I_3 = 3.0$ A entering the junction. Wire 4 carries a current such that 180 C of charge flows out of the junction through it in 1.0 minute. (a) State Kirchhoff's first law and the physical principle it is based on. (b) Calculate the current in wire 4. (c) Determine the magnitude and direction of the current in wire 5.

20. A cell of e.m.f. E and internal resistance r is connected to a variable resistor of resistance R. When $R = 3.0\ \Omega$, the power dissipated in R is 12 W. When $R = 12\ \Omega$, the power dissipated in R is also 12 W. (a) Write down an algebraic expression for the power P dissipated in R in terms of E, r, and R. (b) Use the data provided to determine r. (c) Calculate E. (d) Calculate the maximum power that can be dissipated in R.

11 Particle physics（粒子物理）

Atoms, nuclei, radiation, fundamental particles, and their interactions.

Rules 规则

- Nucleon number A and proton number Z are conserved in all nuclear processes (核反应中核子数和质子数守恒).
- Isotopes are forms of the same element with the same number of protons but different numbers of neutrons (同位素具有相同的质子数和不同的中子数).
- α-particles have discrete energies, while β-particles have a continuous range of energies due to the emission of (anti)neutrinos (α 粒子具有分立能级，β 粒子因发射中微子/反中微子而具有连续能谱).
- Hadrons are composed of quarks: baryons consist of three quarks, and mesons consist of a quark and an antiquark (强子由夸克组成：重子包含三个夸克，介子包含一个夸克和一个反夸克).
- Leptons, such as electrons and neutrinos, are fundamental particles that do not experience the strong nuclear force (轻子，如电子和中微子，是不受强核力作用的基本粒子).

Worked Examples 例题

Example 11.1

A nucleus of ${}^{238}_{92}\mathrm{U}$ decays by emitting an α-particle. Write the complete nuclear equation for this decay.

Solution

The α-particle is represented as ${}^{4}_{2}\alpha$. Conserving the nucleon number: $238 - 4 = 234$. Conserving the proton number: $92 - 2 = 90$. The element with $Z = 90$ is Thorium (Th). The equation is: ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha$

Example 11.2

Determine the quark composition of a π^+ meson, given that it has a charge of $+1e$ and contains an up quark.

Solution

A meson consists of a quark and an antiquark. The up quark (u) has a charge of $+\frac{2}{3}e$. To achieve a total charge of $+1e$, the antiquark must have a charge of $+1e - \frac{2}{3}e = +\frac{1}{3}e$. The down quark (d) has a charge of $-\frac{1}{3}e$, so the anti-down quark ($\bar{\text{d}}$) has a charge of $+\frac{1}{3}e$. The composition is $\text{u}\bar{\text{d}}$.

Example 11.3

Describe the change to the quark composition of a nucleon that occurs during β^- decay.

Solution

During β^- decay, a neutron (^1_0n) decays into a proton (^1_1p). The quark composition of a neutron is udd and a proton is uud. Therefore, one down quark changes into an up quark ($\text{d} \to \text{u}$).

考点总览 Key Points

- □ 我能运用夸克 (quark) 模型解释强子结构，并计算核反应质量亏损与结合能（$E = \Delta mc^2$）
- □ 我理解放射性衰变 (radioactive decay) 的随机性，并能计算活度、半衰期及衰变规律（$A = \lambda N$, $\lambda = \frac{0.693}{t_{1/2}}$）
- □ 我掌握匀速圆周运动 (circular motion) 的角速度、向心加速度及向心力计算（$a = \frac{v^2}{r} = r\omega^2$, $F = \frac{mv^2}{r}$）
- □ 我会运用万有引力定律 (gravitation) 分析天体与同步卫星运动、引力场强及引力势（$g = \frac{GM}{r^2}$, $\phi = -\frac{GM}{r}$）
- □ 我理解热平衡、比热容与潜热，并能运用热力学第一定律 (first law of thermodynamics) 分析内能变化（$\Delta U = q + W$）
- □ 我掌握理想气体 (ideal gas) 状态方程与气体动理论推导，能计算分子平均平动动能（$pV = nRT$, $E_k = \frac{3}{2}kT$）
- □ 我能分析简谐运动 (SHM) 的运动学特征与能量转化，并理解阻尼 (damping) 与共振 (resonance)（$a = -\omega^2 x$）
- □ 我会运用库仑定律 (Coulomb's law) 计算点电荷相互作用力、电场强度及电势（$E = \frac{Q}{4\pi\varepsilon_0 r^2}$, $V = \frac{Q}{4\pi\varepsilon_0 r}$）
- □ 我掌握电容器 (capacitor) 的串并联规律、储能公式及放电指数衰减规律（$C = \frac{Q}{V}$, $W = \frac{1}{2}CV^2$, $\tau = RC$）
- □ 我理解磁感应强度定义与霍尔效应 (Hall effect)，并能运用法拉第电磁感应定律计算感应电动势（$F = BIL\sin\theta$, $\Phi = BA$）
- □ 我掌握交变电流 (alternating current) 的有效值 (r.m.s.) 计算，并熟悉二极管整流与电容滤波原理
- □ 我掌握光电效应 (photoelectric effect) 方程、德布罗意波长及原子能级跃迁辐射（$hf = \Phi + E_{k,\max}$, $\lambda = \frac{h}{p}$）
- □ 我熟悉超声波声阻抗 (acoustic impedance)、X 射线衰减及 PET 扫描中的正负电子湮灭 (annihilation) 机制
- □ 我会利用标准烛光 (standard candle) 测距，并运用哈勃定律 (Hubble's law) 与红移解释宇宙膨胀（$v = H_0 d$）

Basic Drill（入门综合）· 50

1. In an α-particle scattering experiment, a very small fraction of the incident α-particles are deflected through angles greater than 90°. State the conclusion about the structure of the atom deduced from this observation.

2. A neutral atom of uranium is represented by the symbol $^{238}_{92}\mathrm{U}$. Determine the number of neutrons in the nucleus of this atom.

3. State the similarity and the difference between the nuclei of two isotopes of the same chemical element.

4. The elementary charge is $e = 1.60 \times 10^{-19}$ C. Calculate the electric charge of an α-particle in coulombs.

5. The mass of an α-particle is 4.0015 u. Given that 1 u $= 1.66 \times 10^{-27}$ kg, calculate the mass of the α-particle in kilograms to 3 significant figures.

6. A nucleus of radium-226 ($^{226}_{88}$Ra) undergoes α-decay to form a nucleus of radon (Rn) and an α-particle ($^{4}_{2}\alpha$). State the nucleon number A and proton number Z of the resulting radon nucleus.

7. A radioactive nucleus undergoes β^- decay. State the name of the uncharged lepton that is emitted together with the electron in this decay process.

8. A nucleus of sodium-22 ($^{22}_{11}$Na) undergoes β^+ decay to produce a neon (Ne) nucleus, a positron, and an electron neutrino. State the nucleon number A and proton number Z of the neon nucleus.

9. Explain why β-particles emitted from a sample of a single radioactive nuclide are emitted with a continuous distribution of kinetic energies, whereas α-particles are emitted with discrete energies.

10. State the rest mass and electric charge of a positron in terms of the mass m_e and charge e of an electron.

11. From the following list of particles, identify all the fundamental particles: proton, electron, neutron, up quark, neutrino, α-particle.

12. State the electric charge, in terms of the elementary charge e, of: (a) an up (u) quark, (b) a down (d) quark.

13. State the electric charge, in terms of the elementary charge e, of a strange antiquark ($\bar{s}$).

14. In a nuclear reaction, a nucleus of boron-10 ($^{10}_{5}$B) absorbs a neutron ($^{1}_{0}$n) and emits an α-particle ($^{4}_{2}\alpha$) to form a new nucleus Y. Determine the nucleon number A and proton number Z of nucleus Y.

15. A baryon known as the Σ^0 particle consists of one up quark, one down quark, and one strange quark (uds). Determine the total electric charge of the Σ^0 particle in terms of e.

16. A pion π^+ is a meson with the quark composition $u\bar{d}$. Show that the overall charge of the π^+ meson is $+e$.

17. A neutral meson consists of a down quark (d) with electric charge $-\frac{1}{3}e$ and an antiquark. Determine the electric charge, in terms of the elementary charge e, of this antiquark.

18. A nucleus in an excited energy state de-excites by emitting a γ-ray photon. State the change, if any, to the nucleon number and proton number of the nucleus.

19. Distinguish between a baryon and a meson in terms of their quark structure.

20. A nucleus of carbon-12 ($^{12}_{6}$C) consists of 6 protons and 6 neutrons. Calculate the total number of up (u) quarks in this nucleus.

21. A nucleus of lithium-7 ($^{7}_{3}$Li) contains 3 protons and 4 neutrons. Calculate the total electric charge of the lithium-7 nucleus in coulombs. ($e = 1.60 \times 10^{-19}$ C)

22. State the nature of γ-radiation and give its electric charge and mass.

23. An antiproton is the antiparticle of a proton. State the quark composition and the net electric charge, in terms of e, of an antiproton.

24. A nucleus of thorium-234 ($^{234}_{90}$Th) decays by β^- emission to form a nucleus of protactinium (Pa). State the complete nuclear equation for this decay.

25. The nucleus of an oxygen isotope contains 8 protons and 9 neutrons. Write down the symbol for this nuclide in the form $^{A}_{Z}$X.

26. State the quark composition of a neutron in terms of up (u) and down (d) quarks.

27. State the electric charge of an anti-strange antiquark ($\bar{s}$) in terms of the elementary charge e.

28. A stationary radium-226 nucleus ($^{226}_{88}\text{Ra}$) undergoes α-decay to form a radon (Rn) nucleus. State the nucleon number and the proton number of the radon nucleus.

29. State the general name given to a hadron that consists of one quark and one antiquark.

30. State why β-particles emitted in radioactive decay are observed to have a continuous range of energies.

31. A B^- meson consists of a bottom quark (b) and an up antiquark ($\bar{u}$). Calculate the net electric charge of this meson in terms of the elementary charge e.

32. A hypothetical baryon consists of two charm quarks (c) and one strange quark (s). Calculate the total electric charge of this baryon in terms of the elementary charge e.

33. Determine the number of neutrons in a nucleus of uranium-235 ($^{235}_{92}\text{U}$).

34. Identify the neutral fundamental lepton emitted during β^+ decay.

35. In the α-particle scattering experiment, a very small fraction of α-particles were deflected through angles greater than $90°$. State the conclusion drawn about the distribution of mass and positive charge in the atom from this observation.

36. Determine the charge, in terms of e, of a meson with the quark composition $u\bar{d}$.

37. A carbon-12 nucleus ($^{12}_{6}\text{C}$) contains 6 protons and 6 neutrons. Calculate the total number of up quarks (u) and the total number of down quarks (d) in this nucleus.

38. State the mass of an α-particle in unified atomic mass units (u) to the nearest integer.

39. State the change, if any, in (a) nucleon number A and (b) proton number Z of a nucleus when it emits a single β^- particle.

40. A nuclear reaction is represented by $^{14}_{7}\text{N} + ^{4}_{2}\alpha \rightarrow ^{17}_{8}\text{O} + X$. Identify the emitted particle X.

41. State the electric charge, in coulombs, and the mass, in unified atomic mass units (u), of an antiparticle of an electron ($e = 1.60 \times 10^{-19}$ C).

42. List the six flavours (types) of quarks.

43. State whether an electron is a hadron or a lepton, and state whether it is a fundamental particle.

44. Calculate the net charge, in terms of e, of a baryon with the quark structure uds.

45. A nucleus of sodium-22 ($^{22}_{11}$Na) undergoes β^+ decay to form neon (Ne). Write down the nuclide notation $^{A}_{Z}$Ne for the daughter nucleus.

46. Define what is meant by isotopes of an element.

47. A radioactive decay is represented by the equation $^{14}_{6}\mathrm{C} \rightarrow ^{A}_{Z}\mathrm{N} + ^{0}_{-1}\beta + Y$. State the values of A and Z, and identify the particle Y.

48. Calculate the electric charge, in coulombs, of an oxygen-17 nucleus ($^{17}_{8}$O). Take the elementary charge as $e = 1.60 \times 10^{-19}$ C.

49. State the quark composition of an antiproton ($\bar{\mathrm{p}}$).

50. In the α-particle scattering experiment, the vast majority of α-particles pass straight through the gold foil with little or no deflection. State the conclusion about the structure of the atom that is drawn from this observation.

Practice（综合练习）· 25

1. The α-particle scattering experiment provided evidence for the nuclear model of the atom. (a) State the two main observations from the α-particle scattering experiment. (b) Explain how these observations led to the conclusion that the atom has a small, massive, positively charged nucleus.

2. A neutral atom of uranium-235 is represented by the symbol $^{235}_{92}\mathrm{U}$. (a) State the number of protons, neutrons, and orbital electrons in this neutral atom. (b) Determine the total charge of the nucleus of this atom. (c) Uranium has another naturally occurring isotope, uranium-238. Explain what is meant by the term isotope.

3. Plutonium-239 ($^{239}_{94}\mathrm{Pu}$) undergoes α-decay to form an isotope of uranium (U). (a) Write the complete nuclear equation for this decay. (b) State the mass in unified atomic mass units (u) and the charge in terms of elementary charge e of an α-particle. (c) The α-particles emitted from a sample of plutonium-239 have discrete energies. State what is meant by discrete energies.

4. Carbon-14 ($^{14}_{6}\mathrm{C}$) is a radioactive isotope that decays by β^- emission to form nitrogen (N). (a) Write the complete nuclear equation for this decay. (b) State the name of the fundamental particle that is produced and emitted alongside the β^- particle in this process. (c) Explain why the emitted β^- particles have a continuous range of energies.

5. Protons and neutrons are hadrons. (a) State the quark composition of a proton and a neutron. (b) Use the charges of the quarks to show that a neutron has zero total charge. (c) State whether protons and neutrons are fundamental particles.

6. Hadrons can be classified into baryons and mesons. (a) State the quark composition of a baryon and a meson. (b) A particular meson consists of an up quark and an anti-down quark. Determine its total charge in terms of e.

7. Fluorine-18 ($^{18}_{9}\mathrm{F}$) decays by β^+ emission to form an isotope of oxygen (O). (a) Write the complete nuclear equation for this decay. (b) Describe the change to the quark composition of a nucleon that takes place during this β^+ decay.

8. Particles can be classified into different groups, such as hadrons and leptons. (a) State the difference between a hadron and a lepton in terms of fundamental particles. (b) Classify the following particles as hadrons or leptons: electron, neutron, positron, electron neutrino.

9. An antiparticle has the same mass but opposite charge to its corresponding particle. (a) State the name of the antiparticle of an electron. (b) State the mass of this antiparticle in kg. (c) State the name of the antiparticle produced during β^- decay.

10. The unified atomic mass unit (u) is commonly used in nuclear physics. (a) State the value of 1 u in kg. (b) Calculate the mass, in kg, of an α-particle, assuming its mass is exactly 4.00 u. Give your answer to 3 significant figures. (c) Calculate the ratio of the mass of an α-particle to the rest mass of an electron.

11. Radioactive sources can emit α-, β-, and γ-radiations. (a) Compare the composition of an α-particle and a β^--particle. (b) Describe the composition of γ-radiation. (c) State which of these three radiations is not deflected by an electric field.

12. In a nuclear reaction, nucleon number and charge are conserved. A nucleus of americium-241 ($^{241}_{95}$Am) decays by emitting an α-particle to form a nucleus of neptunium (Np). (a) Write the nuclear equation for this decay. (b) Show that charge is conserved in this decay.

13. A nucleus of thorium-232 ($^{232}_{90}$Th) undergoes a series of radioactive decays to become a stable nucleus of lead-208 ($^{208}_{82}$Pb). The decays involve only α and β^- emissions. (a) Calculate the total change in nucleon number and hence determine the number of α-particles emitted. (b) Determine the number of β^--particles emitted in this decay chain.

14. There are six flavours of quarks. (a) Name the six flavours of quarks. (b) State the charge, in terms of e, of a top quark and a bottom quark. (c) An anti-charm quark has a charge of $-2/3\ e$. Deduce the charge of a charm quark.

15. During β^- decay, a neutron decays into a proton. (a) Write an equation for this process in terms of quarks. (b) Explain how charge is conserved in this quark transformation.

16. In the α-particle scattering experiment, a beam of α-particles is directed at a thin gold foil. (a) State the observation that indicates the nucleus is very small compared to the size of the atom. (b) Explain how the observation that a small fraction of α-particles are deflected by angles greater than 90° supports the nuclear model of the atom.

17. A stationary nucleus of radium-226 ($^{226}_{88}$Ra) decays into radon (Rn) and an α-particle. (a) Write the nuclear equation for this decay. (b) State the nucleon number and proton number of the radon nucleus.

18. A pion (π^+) is a meson with a charge of $+e$. It is composed of an up quark and one antiquark. (a) State the charge of an up quark. (b) Deduce the flavour of the antiquark in the π^+ meson. (c) State whether the π^+ meson is a fundamental particle.

19. A proton can decay into a neutron within a nucleus via β^+ decay. (a) State the name of the two particles emitted in this process. (b) Write the nuclear equation for the β^+ decay of carbon-11 ($^{11}_{6}$C) to boron (B).

20. The ratio of charge to mass of a particle is a useful property in physics. (a) Calculate the ratio of charge to mass of an α-particle. Assume the mass of the α-particle is 4.00 u. Give your answer to 3 significant figures. (b) Calculate the ratio of charge to mass of a β^--particle. Give your answer to 3 significant figures.

21. Hydrogen has three isotopes: hydrogen-1 ($^{1}_{1}$H), hydrogen-2 ($^{2}_{1}$H), and hydrogen-3 ($^{3}_{1}$H). (a) Describe the similarities and differences in the nuclear composition of these three isotopes. (b) Hydrogen-3 (tritium) is radioactive and undergoes β^- decay to helium (He). Write the equation for this decay.

22. The existence of the neutrino was proposed to explain the continuous energy spectrum of β-particles in radioactive decay. (a) Explain what is meant by a continuous energy spectrum. (b) Explain how the emission of an antineutrino accounts for this continuous range of energies in β^- decay.

23. A Δ^{++} particle is a baryon with a charge of $+2e$. (a) State the number of quarks in a baryon. (b) Deduce the quark composition of the Δ^{++} particle.

24. A kaon (K^-) is a meson containing a strange quark (s) and an anti-up quark ($\bar{u}$). The charge of the strange quark is $-1/3$ e. (a) Verify that the total charge of the K^- meson is $-e$. (b) State the quark composition of the K^+ meson, which is the antiparticle of the K^- meson. (c) Determine the charge of the K^+ meson.

25. Magnesium-23 ($^{23}_{12}\text{Mg}$) undergoes β^+ decay to sodium (Na). (a) Write the complete nuclear equation for this decay. (b) State the class of fundamental particles to which the emitted β^+ particle and neutrino belong. (c) Determine the total number of up quarks and down quarks in a nucleus of magnesium-23.

Advanced Practice（培优）· 20

1. A doubly ionised atom of an isotope of zinc ($^{66}_{30}$Zn) has a charge of $+2e$. Determine the ratio of charge to mass for this ion in C kg^{-1}. Use 1 u $= 1.66 \times 10^{-27}$ kg and assume the mass of the ion is approximately equal to its nucleon number in u.

2. In the α-particle scattering experiment, a narrow beam of α-particles is directed at a thin gold foil in a vacuum. (a) State why the experiment must be conducted in a vacuum. (b) State the observation from this experiment that leads to the conclusion that the nucleus represents most of the mass of the atom and is positively charged.



Fig. 2.

3. A stationary nucleus of bismuth-212 ($^{212}_{83}$Bi) decays into thallium-208 ($^{208}_{81}$Tl) by emitting an α-particle. (a) State the composition of an α-particle. (b) Explain why the emitted α-particles have discrete energies, whereas β-particles emitted in radioactive decay have a continuous range of energies.

4. A baryon consists of one up quark, one strange quark and one top quark. (a) Deduce the total charge of this baryon in terms of the elementary charge e. (b) State whether this baryon is a fundamental particle, and explain your answer.

5. A meson is composed of a charm quark and an anti-strange quark. (a) Determine the charge of this meson in terms of e. (b) The meson decays to produce a positron and another fundamental particle. State the name of the group of fundamental particles to which the positron belongs, and identify the other fundamental particle produced.

6. The nuclide carbon-14 ($^{14}_{6}$C) decays by β^- emission to a stable isotope of nitrogen (N). (a) Write the complete nuclear decay equation for this process. (b) Describe the precise change to the quark composition of a single nucleon that occurs during this decay.

7. An antiproton is the antiparticle of a proton. (a) State the quark composition of an antiproton. (b) Calculate the rest mass of an antiproton in kg, given that the mass of a proton is 1.007 u and 1 u $= 1.66 \times 10^{-27}$ kg. Give your answer to 3 significant figures.

8. A sample of a radioactive isotope emits both β^+ particles and γ-rays. (a) Compare the mass and charge of a β^+ particle with that of an electron. (b) State the difference between a β^+ particle and a γ-ray in terms of their fundamental physical nature.

9. An isotope of uranium, $^{235}_{92}$U, undergoes a radioactive decay chain, emitting 7 α-particles and 4 β^--particles to become a stable isotope of lead (Pb). Determine the nucleon number and proton number of this stable lead isotope.

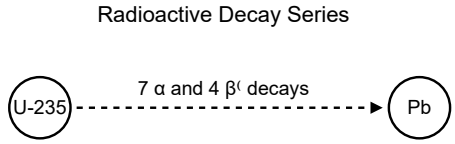

Fig. 9.

10. In a nuclear reaction, a nucleus of aluminium-27 ($^{27}_{13}$Al) absorbs an α-particle and subsequently emits a neutron to form a new nuclide Y. Determine the number of protons and neutrons in a nucleus of Y.

11. Fluorine-18 ($^{18}_{9}$F) decays into oxygen-18 ($^{18}_{8}$O) by emitting a β^+ particle. (a) Write the complete nuclear equation for this decay. (b) Calculate the total number of up quarks and down quarks in an oxygen-18 nucleus. (A proton is *uud* and a neutron is *udd*.)

12. A fully ionised atomic nucleus has a mass of 1.162×10^{-26} kg and a charge of $+4.80 \times 10^{-19}$ C. (a) Calculate the nucleon number of this nucleus, using 1 u $= 1.66 \times 10^{-27}$ kg. (b) Determine the number of neutrons in this nucleus.

13. The Σ^+ baryon has a charge of $+e$ and contains two up quarks and one other quark q. (a) Identify the possible flavours for quark q. (b) State the difference between a baryon and a meson in terms of their quark composition.

14. An atom of gold ($^{197}_{79}$Au) is electrically neutral. (a) State the number of protons, neutrons, and electrons in this atom. (b) The radius of the gold nucleus is about 10^{-4} times the radius of the gold atom. Estimate the ratio of the volume of the atom to the volume of its nucleus, and state what this implies about the structure of the atom.

15. Calculate the total charge, in coulombs, of all the up quarks present in a single α-particle. Give your answer to 2 significant figures.

16. When a free neutron decays into a proton, a β^- particle is emitted. (a) State the name of the other fundamental particle emitted in this process. (b) Explain how the emission of this other particle accounts for the continuous range of energies observed for the emitted β^- particles.

17. A positron and a proton both have a charge of $+e$. (a) State the quark composition of a proton, and explain why a positron cannot be described in terms of quarks. (b) State the name of the particle that is the antiparticle of the proton, and deduce its charge.

18. A nuclide Z has a nucleon number A and proton number P. It undergoes one α decay followed by two successive β^- decays. (a) Deduce the nucleon number and proton number of the final nuclide in terms of A and P. (b) State the relationship between the original nuclide Z and the final nuclide.

19. The unified atomic mass unit, u, is defined as 1/12 of the mass of a carbon-12 ($^{12}_{6}$C) atom. Calculate the mass of a carbon-12 atom in kg, and estimate the mass of the nucleus of a carbon-12 atom by subtracting the mass of its electrons. State the mass of the nucleus to 5 significant figures. (1 u = 1.6605×10^{-27} kg, mass of an electron = 9.11×10^{-31} kg).

20. Consider the following particles: electron, electron antineutrino, down quark, and positron. (a) Identify which of these particles are classified as leptons. (b) Determine the ratio of charge magnitude to rest mass for a positron, in C kg^{-1}. (mass of positron = mass of electron = 9.11×10^{-31} kg).